

## ***Interactive comment on “Height of convective layer in planetary atmospheres with condensable and non-condensable greenhouse substances” by A. M. Makarieva et al.***

**A. M. Makarieva et al.**

Received and published: 4 March 2004

Here we address in detail the point made twice by Referee 2 under No. 1 in his first and second commentaries. The referee objected the use of linear Eddington's approximation (2.1) by noting that at large optical depths the outgoing flux is not dependent on surface emission but on atmospheric emission and suggested to check the  $k$  distribution model for about half a number of  $k$ .

In this commentary we show that greenhouse effect is determined by diffusion (random walk) of thermal photons in the atmosphere. This process corresponds to a  $\tau_s$ -linear dependence ( $\tau_s$  is the atmospheric optical thickness) between fluxes of thermal radiation at the surface,  $F_s$ , and outgoing radiation,  $F_e$ , at any frequency  $\nu$  of the thermal spectrum. Greenhouse effect cannot be described with use of Lambert's law (the ex-

S2614

ponential drop of radiative flux with growing optical depth  $\tau$ ) in any approximation and at any intermediate stages of calculations.

Due to the linear relationship between  $F_s$  and  $F_e$  the dependence of the greenhouse effect on concentrations of atmospheric greenhouse substances has a linear-fractional form and does not contain either a square root or logarithmic dependence on the concentrations at any, including large (corresponding to  $\tau \gg 1$ ), values of these concentrations. The linear, square root and logarithmic dependencies of the absorption coefficients on concentration (which include the corresponding dependencies on temperature and pressure and are derived from Lambert's law for each frequency) only arise in spectroscopic laboratory and atmospheric experiments. In such experiments the fundamental parameters of molecular absorption cross-sections  $\sigma$  and atmospheric optical thickness  $\tau_s$  can be empirically determined with use of solar or laser (not Earth's thermal!) radiation.

The above statements are very well-known in both radiative transfer and greenhouse effect theory as well as in atmospheric spectroscopy. Various misunderstandings arise at the interface of spectroscopic and greenhouse effect studies, which we illustrate and clarify below.

### **1. Atmospheric spectroscopy**

With use of molecular spectroscopy it is possible to measure the fundamental value of the cross-section  $\sigma_\nu$  [cm<sup>2</sup>] which describes the process of absorption of photons with frequency  $\nu$  [sec<sup>-1</sup>] by the corresponding greenhouse substance.

Importantly, such measurements are performed using sources of radiation that are characterised by **brightness temperature much higher than the temperature of the medium** (e.g., solar radiation or lasers). When such radiative fluxes interact with the matter, radiation is **irreversibly** removed from the radiative field. Due to this fact radiative flux diminishes exponentially in accordance with Lambert's law,  $F_\nu = F_{\nu 0} e^{-\tau_\nu}$ , where  $\tau_\nu = \sigma_\nu n h$ ,  $n$  is the mean concentration of the absorber's molecules over length

S2615

$h$  along the ray's path.

Introducing absorption coefficient  $k_\nu \equiv \tau_\nu/m$ , where  $m \equiv m_{mol}nh$  ( $m_{mol}$  is mass of one molecule), the transmissivity function can be written as  $e^{-\tau_\nu} = e^{-k_\nu m}$ . When there is a complex system of overlapping lines the transmissivity function can be represented with use of Laplace transformation in the form  $\int_0^\infty e^{-km} f(k) dk$  ( $k$ -distribution). There are also many other ways of describing the mean transmissivity function (see, e.g., Goody and Yung, 1989, pp. 145-177), all of them based on and starting from the above Lambert's exponent for  $F_\nu$ .

Relative absorptivity for a given frequency is  $(F_{\nu 0} - F_\nu)/F_{\nu 0} = 1 - e^{-\tau_\nu}$ , while mean transmissivity is  $e^{-\tau_\nu}$ . When Lambert's exponent is superimposed on the line absorption profile, the contribution of the line centre (where  $\sigma_\nu$  and  $\tau_\nu$  are maximum) into the mean transmissivity function is suppressed as compared to line wings (where  $\sigma_\nu$  and  $\tau_\nu$  are smaller). Due to the dependence of line width on pressure, at different values of  $\tau_\nu$  and  $m$  this yields different dependencies of the mean transmissivity on concentrations of the greenhouse substances, starting from the linear function at small  $\tau$ , then according to the square root, and, finally, to the logarithm of concentrations (see, e.g., Goody and Yung, 1989, pp. 125-187).

When at large  $\tau_\nu$  the second term  $e^{-\tau_\nu}$  in the absorptivity function  $(F_{\nu 0} - F_\nu)/F_{\nu 0} = 1 - e^{-\tau_\nu}$  becomes indistinguishable from zero within the accuracy of the experiment, one says that absorption is saturated.

All the above constitutes the well-known bases of radiative spectroscopy, but is largely irrelevant to the description of the dependence of the planetary greenhouse effect on concentrations of atmospheric substances.

## 2. Greenhouse effect

### 2.1 Lambert's law neglects local emission

The greenhouse effect is analysed with use of the radiative transfer equation repre-

S2616

sented as (Goody and Yung, 1989, p. 22):

$$l_\nu \frac{dI_\nu(P, \varepsilon)}{ds} = -I_\nu(P, \varepsilon) + J_\nu(P, \varepsilon), \quad \frac{d}{ds} \equiv \mu_i \frac{\partial}{\partial x_i} = \varepsilon \cdot \nabla, \quad l_\nu^{-1} = \sigma_\nu n \equiv e_{v,\nu}. \quad (1)$$

Here  $n$  [molecules  $\text{cm}^{-3}$ ] is the density of the greenhouse substance;  $l$  [cm] is the mean free path length of thermal photons;  $e_{v,\nu}$  [ $\text{cm}^{-1}$ ] is the volume absorption coefficient (as denoted by Goody and Yung, 1989, p. 22);  $\frac{d}{ds}$  ( $\text{cm}^{-1}$ ) is the gradient of changes in the direction of the unit vector  $\varepsilon \equiv \frac{\mathbf{s}}{s}$ ;  $\mu_i$  is the cosine of the angle between the chosen direction of intensity  $\varepsilon$  and  $i = x, y, z$  directions.

Eq. (1) represents the energy conservation law for the local volume: the change in radiation intensity  $I_\nu(P, \varepsilon)$  in unit solid angle along  $\varepsilon$  over free path length  $l$  is equal to the difference between absorption and emission of radiation by the medium. Absorption is assumed to be proportional to intensity itself, which is a consequence of the smallness of light-on-light scattering.

It is important to underline that Lambert's law is not a consequence of the local proportionality between absorption and intensity. Lambert's law can be obtained from Eq. (1) **if and only if** local emission  $J_\nu(P, \varepsilon)$  is negligible in comparison to local absorption and intensity, i.e. when  $I_\nu(P, \varepsilon) \gg J_\nu(P, \varepsilon)$ . (In particular, only in this case one can neglect the angular distribution of the re-emitted radiation, so that the chosen direction of the ray's propagation is **conserved over many free path lengths**  $l$ ).

The condition  $I_\nu(P, \varepsilon) \gg J_\nu(P, \varepsilon)$  is only valid for radiation with brightness temperature much higher than the local temperature of the medium (that is, the temperature of thermal radiation). It is valid, in particular, for short-wave solar radiation and for powerful laser beams at all frequencies (including infrared). After interaction with the matter, radiation is irreversibly removed from such radiative fluxes due to the fact that these fluxes have much lower entropy than the natural thermal radiation. This results in an exponential drop of such powerful radiative fluxes as they pass through the medium

S2617

heating it.

## 2.2 Transmissivity function for thermal radiation

An entirely different situation is observed when local emission is of the order of the intensity of the considered radiation,  $I_\nu(P, \varepsilon) \sim J_\nu(P, \varepsilon)$ , as is the case for thermal radiation. Photons of natural thermal radiation that is characterised by maximum entropy **cannot disappear** after their interaction with the matter. Neither, unlike laser and solar radiation, can thermal photons perform net transfer of energy to the matter thus heating it. On the contrary, **it is the kinetic energy of matter heated during dissipation of non-radiative convective energy fluxes which is converted into thermal radiation during collisions of photons and molecules.** Therefore brightness temperature of the natural thermal radiation is lower than air temperature, which was used in our paper.

In the absence of convection and other non-radiative energy fluxes, there arises the state of radiative equilibrium first described by Schwarzschild, when there is no net energy exchange between matter and radiation. (Note that in this case air temperature is **constant at all heights and coincides with surface temperature**). Under conditions of radiative equilibrium the non-disappearing thermal photons are absorbed and then re-emitted by the medium in an arbitrary direction. The resulting pattern of propagation of thermal photons in the atmosphere is the random walk pattern. As is well-known, random walk represents a diffusion process which corresponds to the **linear** drop of concentration of thermal photons (i.e. radiative energy density and thermal radiative flux) with decreasing optical depth  $\tau$ . The corresponding linear relationship for thermal radiative flux is very well-known and is called Eddington's approximation,  $F(\tau) = F_e(1 + c\tau)$  and  $F_s = F_e(1 + c\tau_s)$ , where  $c$  is a geometric constant and  $\tau_s$  is the optical thickness of the atmosphere (see, for instance, Michalas and Michalas, 1984; Gorshkov and Makarieva, 2002).

Accordingly, the transmissivity function  $F_e/F_s$  for thermal radiation does not contain

S2618

any traces of Lambert's exponent  $e^{-\tau_s}$ , but is proportional to  $1/(1 + c\tau_s)$ . The relative greenhouse effect which can be defined as  $(F_s - F_e)/F_e$  and describes the number of times the surface radiative flux  $F_s$  has grown as compared to thermal radiative flux into space  $F_e$  due to the presence of atmospheric greenhouse substances is directly proportional to  $\tau_s$ , i.e. to the total amount of greenhouse substances in the atmosphere.

There cannot be any saturation of the greenhouse effect in the centre of absorption lines. This misleading idea is based on Lambert's exponent and confusion of spectroscopic measurements employing lasers and thermal radiation in the real atmosphere. Such a logic implies that, were there no pressure and Doppler broadening, terrestrial greenhouse effect would remain unaffected by growing CO<sub>2</sub> concentration, as far as absorption in the centre of the 15  $\mu m$  band is already saturated at present values of CO<sub>2</sub> partial pressure and air pressure (see IPCC 1995, Radiative forcing, p. 174). However, the greenhouse effect changes primarily not due to pressure broadening (which, in the general case, may or may not be present dependent on the absolute magnitude of concentrations of the atmospheric gases). The greenhouse effect grows primarily due to the growing **total amount** of greenhouse substances in the atmosphere, as described by Eddington's approximation (Michalas and Michalas, 1984). (This growth would, for example, take place even in a hypothetical atmosphere which would expand in space with growing  $\tau_s$  in such a manner that local concentrations of atmospheric substances remain the same, thus prohibiting pressure broadening.)

In our paper we have shown that the account of convection does not change the general character of the random walk of thermal photons in the atmosphere, but results in a decrease of the gradient of radiative energy density in the region of convection. With growing atmospheric optical thickness the relative contribution of convection into the greenhouse effect decreases, while the accuracy of Eddington's approximation for its description increases.

## 2.3 The mathematics

S2619

Let us now discuss the mathematics which is used to quantify the radiative transfer process.

Directing  $\varepsilon = \frac{s}{s}$  along the  $z$  axis one can write Eq. (1) as (Goody and Yung, 1989, p. 43):

$$\frac{dI_\nu(\tau)}{d\tau} = I_\nu(\tau) - J_\nu(\tau), \quad \tau \equiv \int_z^\infty \frac{dz}{l(z)}, \quad l(z) \equiv \frac{1}{\sigma_\nu n(z)}. \quad (2)$$

Multiplying both parts of the equation by  $e^{-\tau}$ , moving the first term of the right-hand part into the left-hand part of the equation and integrating both parts of the equation thus obtained within the limits  $\tau = 0$  and  $\tau = \tau_s$ , one can represent the differential equation thus obtained in the integral form:

$$I_\nu(0) = I_\nu(\tau_s)e^{-\tau_s} + \int_0^{\tau_s} J_\nu(\tau)e^{-\tau} d\tau. \quad (3)$$

**If and only if**  $J_\nu(\tau)$  had been an external function independent on  $I_\nu(\tau)$ , Eq. (3) would have represented an integral solution of Eq. (2). In this case Eq. (3) could have been interpreted as follows.

Intensity  $I_\nu(0)$  of thermal radiation at  $\tau = 0$  is equal to the sum of intensities of radiation emitted within local volumes and weakened exponentially as prescribed by Lambert's law in accordance with their optical distance from the top of the atmosphere. In such an interpretation, due to the exponential drop of radiation intensity the main contribution into  $I_\nu(0)$  is made by the upper atmospheric layers, that is, is dependent on atmospheric rather than surface emission, the statement insisted upon by Referee No. 2. **If and only if** the radiation emission  $J_\nu$  within these layers would have been independent of surface radiation, the outgoing flux of thermal radiation would have been only very weakly dependent on surface radiation.

Integrating the integral part in Eq. (3) by parts and taking into account that at the Earth's

S2620

surface the source function coincides with radiation of the Earth's surface,  $J_\nu(\tau_s) = I_\nu(\tau_s)$ , while outside the atmosphere at  $\tau = 0$  the source function is equal to zero,  $J_\nu(0) = 0$ , it is possible to represent Eq. (3) in the form containing absorptivity  $1 - e^{-\tau}$  instead of transmissivity  $e^{-\tau}$  (see, e.g., Raval, Ramanathan, 1989, p. 759):

$$I_\nu(0) = I_\nu(\tau_s) - \int_0^{\tau_s} (1 - e^{-\tau}) \left( \frac{dJ_\nu(\tau)}{d\tau} \right) d\tau. \quad (4)$$

Note that using the relationship  $\frac{\tau}{\tau_s} = \frac{p}{p_s} \equiv x$  [derived in our paper] the integral in Eq. (4) can be brought into the form  $\int_0^1 A_\nu(x) \frac{dJ_\nu(x)}{dx} dx$ , where  $A_\nu(x) = \tau_s(1 - e^{-\tau_s x})$ . In this form it was used in the work of Raval and Ramanathan (1989) and in other publications.

According to Eq. (4) the greenhouse effect  $G_\nu$  is determined by the difference (Raval and Ramanathan, 1989, p. 759):

$$G_\nu \equiv I_\nu(\tau_s) - I_\nu(0) = \int_0^1 A_\nu(x) \frac{dJ_\nu(x)}{dx} dx. \quad (5)$$

After integrating Eq. (5) over  $\nu$  we can use all standard spectroscopic methods to determine how the greenhouse effect depends on concentrations of greenhouse substances and air pressure.

We now discuss the above results yielded from Eq. (3) under the assumption of  $J_\nu(\tau)$  **prescribed as an independent external field** on the example of Eq. (3).

At sufficiently large optical thickness  $\tau_s$  absorption in the line centre becomes saturated, so that line centre ceases to give any appreciable contribution into the first term of the right-hand side of Eq. (3). That is, the dependence on intensity of surface radiation vanishes from Eq. (3). After integrating over the absorption profile and thermal spectrum  $I(0)$  is equal to the intensity of the outgoing thermal radiation, which, in the

S2621

stationary case, is equal to solar radiation – that is, does not change with growing  $\tau_s$ . Due to the saturation of absorption in the line centre, the main contribution into the changing absorptivity comes from line wings, which broaden with growing  $\tau_s$  in accordance with spectroscopically determined regularities (square root, then logarithmic dependence). Namely this effect is assumed to be responsible for the presumed weak dependence of  $I(\tau_s)$  on  $\tau_s$  and, consequently, concentrations of greenhouse substances.

At sufficiently large  $\tau_s$  saturation will occur over the entire line profile. Moreover, absorption of so-called grey substances (like clouds) that absorb and emit thermal radiation rather evenly over entire spectrum can be considered fully "saturated" from the very beginning. Using the above logic, at sufficiently large  $\tau_s$  the term  $I(\tau_s)$  cancels from Eq. (3), so that greenhouse effect ceases to depend on  $\tau_s$  (and can never depend on it in case of "saturated" grey substances).

It is highly surprising that this unphysical conclusion and the underlying logic, widely spread as it is (IPCC, 1995, p. 174), did not cause perplexion and concern in the scientific community but were uncritically adopted. These results make null and void all the fundamental achievements made in radiative transfer physics in the beginning of the 20th century, in particular, by Schwarzschild and Eddington.

The greenhouse effect growing infinitely **proportionally** to  $\tau_s$  constitutes the basis of the Universe energetics. Star interiors become nearly infinitely hot (and this leads to the origination of thermonuclear reactions) namely due to the fact that radiative flux infinitely grows proportionally to the inwardly increasing optical depth. The greenhouse effect on Venus where concentration of greenhouse substances is hundreds of times larger than on Earth is likewise hundred of times more powerful than on Earth, in proportion to  $\tau_s$ , not to its logarithm!

The fundamental error in the current usage of Eq. (3) is the assumption that the source function can be prescribed independently of radiation intensity, while in fact  $J$  is an

S2622

approximately linear function of  $I$ , and Eq. (3) is not a solution of the radiative transfer problem.

What is the nature of the source function  $J_\nu(\tau)$  and how it is related to radiation intensity? In the case of radiative equilibrium, as noted above, the propagation of thermal radiation is completely dictated by diffusion processes. In accordance with the diffusion equation, the left-hand part of Eq. (2) is equal to a constant magnitude:

$$\frac{dI_\nu(\tau)}{d\tau} = I_{\nu 0}, \quad J_\nu(\tau) = I_\nu(\tau) - I_{\nu 0}, \quad I_{\nu 0} = \text{const}. \quad (6)$$

Then Eq. (2) is solved as

$$I_\nu(0) = I_\nu(\tau_s) - \tau_s I_{\nu 0}, \quad I_{\nu 0} = I_\nu(0) \cdot c, \quad c = \text{const}, \quad (7)$$

which has the form of Eddington's approximation. (We note for clarity sake that formula (3) can be obtained from Eq. (7) by making a formal, physically meaningless substitution  $I_{\nu 0} \equiv I_\nu(\tau) - J_\nu(\tau)$ ).

Here we do not dwell on the derivation of Eq. (6), which can be found elsewhere (see, e.g., Michalas and Michalas, 1984, p. 357; Gorshkov and Makarieva, 2002). After account of convection, as is shown in our paper currently under discussion, relationships (6) and (7) retain their nature, and it is only the constant  $c$  that somewhat changes its magnitude.

What is then left from the above interpretation of the integral relationship (3) containing the exponential dampening of contributions from different layers? Putting Eq. (6) into Eq. (3) and integrating by parts the term containing  $I_\nu(\tau)$  we obtain

$$\int_0^{\tau_s} \frac{dI_\nu(\tau)}{d\tau} e^{-\tau} d\tau = I_{\nu 0} (1 - e^{-\tau_s}). \quad (8)$$

S2623

Differentiating both parts of (6) over  $\tau_s$  we arrive at Eq. (6). Thus, the exponents in the two terms of the right-hand side of Eq. (3) cancel each other. In other words, in the case of thermal radiation these exponents represent an unlucky mathematical transformation of the initial Eq. (2). This mathematical transformation does not have any physical meaning.

It is common to account for the non-radiative convective energy fluxes using the assumption of local thermodynamic equilibrium (LTE), that is, substituting the source function  $\pi J_\nu(\tau)$  by blackbody radiation  $\sigma_B T^4$  at local air temperature  $T$ . Such a substitution is made irrespective of the magnitude of the non-radiative fluxes as compared to thermal fluxes in the atmosphere, including the case of radiative equilibrium. The LTE assumption, first made by Schwarzschild (1906) **for the analysis of stars** is valid only at large values of optical depth  $\tau$ , when  $\tau I_{0\nu} \gg I_\nu(0)$ , that is, when  $J_\nu(\tau) \approx I_\nu(\tau)$ . Indeed, in such a case brightness temperature  $T_b$  determined by intensity  $I_\nu(\tau) = \sigma_B T_b^4 / \pi$  coincides with air temperature  $T$ . The latter is assumed to be related to the source function as  $J_\nu(\tau) = \sigma T^4 / \pi$ . That is, one has  $T_b \approx T$  and  $J_\nu(\tau) \approx I_\nu(\tau)$  is fulfilled. However, at  $\tau \sim 1$  the assumption of local thermodynamic equilibrium violates the first and second laws of thermodynamics (manifested as temperature discontinuity at the surface (Weaver and Ramanathan, 1995) and non-zero height of convection at  $\tau_s \rightarrow 0$  (Goody and Yung, 1989)). We mentioned this flaw of the LTE assumption in our response to the referee of our previous paper in ACPD (Gorshkov and Makarieva, 2002b). If one uses Eq. (3) and the LTE assumption, due to the exponential drop of radiative flux, the major contribution into the outgoing flux is made namely by the area with  $\tau \sim 1$ , where the LTE assumption is invalid. This is one source of errors in calculating the greenhouse effect with use of Eq. (3).

However, as already noted, the main fault with using Eq. (3) is the misleading idea that the source function can be prescribed as an external field (e.g. via local air temperature) independent of radiation intensity. In reality in the case of thermal radiation and diffusional process of propagation of thermal photons source function is strictly related

S2624

to radiation intensity. For instance, in a planar (stratified atmosphere) with  $z$  directed along the vertical axis and isotropic  $J(z)$  we have in Eq. (1)  $\frac{d}{ds} = \mu \frac{d}{dz}$ . Taking into account that  $\frac{d}{dz} I(z, \mu)$  has no discontinuity and putting  $\mu = 0$ , we obtain an important relation  $J(z) = I(z, 0)$  (from which Eddington's approximation can be easily obtained), showing that the source function cannot be arbitrarily specified! The strict relationship between radiation intensity and source function completely changes the form of Eq. (3) making radiative flux at the surface grow linearly with  $\tau_s$ .

---

Interactive comment on Atmos. Chem. Phys. Discuss., 3, 6701, 2003.

S2625