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A critique of some modern applications of the Carnot heat engine concept: the dissipative heat engine cannot exist

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In several recent studies, a heat engine operating on the basis of the Carnot cycle is considered, where the mechanical work performed by the engine is dissipated within the engine at the temperature of the warmer isotherm and the resulting heat is added to the engine together with an external heat input. This internal dissipation is supposed to increase the total heat input to the engine and elevate the amount of mechanical work produced by the engine per cycle. Here it is argued that such a dissipative heat engine violates the laws of thermodynamics. The existing physical models employing the dissipative heat engine concept, in particular the heat engine model of hurricane development, need to be revised.

Keywords: dissipative heat engine; Carnot cycle; dissipation; efficiency

1. Introduction

The Carnot cycle does not involve irreversible processes or dissipative losses. During one cycle in a Carnot heat engine, the working body (a fluid capable of expansion, usually gas) receives heat Q_{h} from a hot body (the heater) and performs mechanical work $A_{\mathrm{h}} = Q_{\mathrm{h}}$ at temperature T_{h}. It gives heat Q_{c} away to a cold body (the cooler) while work $A_{\mathrm{c}} = Q_{\mathrm{c}} \leq Q_{\mathrm{h}}$ is performed on the gas at temperature $T_{\mathrm{c}} \leq T_{\mathrm{h}}$. The resulting work $A > 0$ is determined by the energy conservation law (the first law of thermodynamics) as $A = Q_{\mathrm{h}} - Q_{\mathrm{c}}$. This work is performed by the working body on its environment. Since all the processes in the Carnot cycle are reversible, entropy of the working body is conserved. Entropy of the environment on which the work is performed does not change either. The amount of entropy $S_{\mathrm{h}} = Q_{\mathrm{h}}/T_{\mathrm{h}}$ received from the heater is equal to the amount of entropy $S_{\mathrm{c}} = Q_{\mathrm{c}}/T_{\mathrm{c}}$ given away to the cooler. The equality $Q_{\mathrm{h}}/T_{\mathrm{h}} = Q_{\mathrm{c}}/T_{\mathrm{c}}$, which stems from the second law of thermodynamics,

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combines with the energy conservation law to determine efficiency $\varepsilon \equiv A/Q_h$ of the Carnot cycle as $\varepsilon = (T_h - T_c)/T_h < 1$. For the Carnot heat engine, $A \le Q_h$.

The dissipative heat engine concept advanced by Rennó & Ingersoll (1996) and discussed by Rennó (1997, 2001), Pauluis *et al.* (2000) and Pauluis & Held (2002) is used to account for hurricane intensity (Bister & Emanuel 1998; Emanuel 2003). In the dissipative heat engine, work A_d (or some part of it in the general case) produced in the ideal Carnot cycle undergoes dissipation at temperature T_h of the heater. (Subscript index d stands for the dissipative heat engine.) The resulting heat is added to the working body together with the external heat Q_{hd} that comes from the heater. In the stationary case, the relationship between work A_d and external heat Q_{hd} is then written as $A_d = \varepsilon(Q_{hd} + A_d)$. Efficiency $\varepsilon_d = A_d/Q_{hd}$ of the dissipative heat engine becomes $\varepsilon_d = (T_h - T_c)/T_c$. Thus, for a given Q_{hd}, the efficiency of the dissipative heat engine grows infinitely with decreasing T_c, and work A_d can become much larger than Q_{hd}: $A_d \gg Q_{hd}$ at $T_c \ll T_h - T_c$ and $\varepsilon_d \gg 1$; $A_d \to \infty$ at $T_c \to 0$. Demanding energy to be conserved gives $Q_{hd} = Q_c$, i.e. the amount of heat received by the dissipative heat engine from the heater coincides with the amount of heat disposed to the cooler. It is assumed that when work A_d dissipates within the working body of the dissipative heat engine in contact with the heater, i.e. at $T = T_h$, entropy increases by $S_{hd} = (Q_{hd} + A_d)/T_h$. The decrease in entropy due to contact with the cooler remains $S_c = Q_c/T_c$ as in the Carnot heat engine. Taking into account that $Q_h = Q_c$ and $A_d = \varepsilon_d Q_{hd}$, the mathematical equality $S_{hd} = S_c$ holds. From this, it is concluded that in the dissipative heat engine the entropy of the working body remains constant and that the dissipative heat engine conforms to both first and second laws of thermodynamics.

The main feature of the dissipative heat engine is the increase in the work produced by the engine per cycle due to internal dissipation compared with the same engine without dissipation. It is stated that 'the fraction of mechanical energy dissipated ... increases the heat input to the convective heat engine' (Rennó & Ingersoll 1996, p. 579), so that 'more energy is available to be converted into mechanical energy' (Rennó & Ingersoll 1996, p. 578). In other words, by dissipating work within the engine, it is supposed to be possible to increase the per-cycle output of the mechanical work. In this paper, we show that this concept of the dissipative heat engine is based on a physical misinterpretation of the nature of the Carnot heat engine. When the essential physical features of the heat engine are taken into account, the concept of the dissipative heat engine is shown to be in conflict with the laws of thermodynamics.

2. Physics of the Carnot heat engine

The Carnot cycle consists of two isotherms at temperatures $T = T_h$ and $T = T_c$ of the heater and the cooler, respectively, and of two adiabates connecting the isotherms. The working body in thermodynamic equilibrium with the heater at $T = T_h$ cannot receive heat from the latter. To receive heat, the working body must expand first, so that its temperature becomes a little lower than that of the heater, only then the heat flux from the heater to the working body becomes

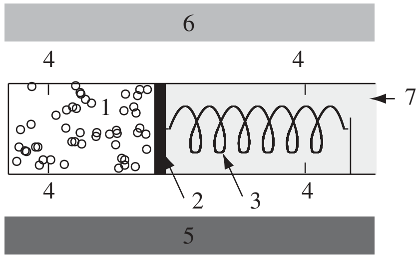

Figure 1. A Carnot heat engine. The working body (gas) (open circles) is contained within a closed cylinder (1) with a sliding piston (2) as the cap. Spring (3) makes the piston move. Elastic stops (4) limit the minimum and maximum volumes of the gas in the cycle. During the cycle, the working body is sequentially brought into contact with the heater (5) at $T = T_\mathrm{h}$ and the cooler (6) at $T = T_\mathrm{c}$. The environment where the engine works (7) is assumed to be infinite, so its pressure p is constant during the cycle.

possible. Thus, the Carnot heat engine operating on the basis of the Carnot cycle must be furnished with an auxilliary dynamic device that performs mechanical expansion and contraction of the working body. A Carnot heat engine where the role of such a device is played by an ideal elastic spring is shown in figure 1. The working body (gas) is contained in the cylinder capped on one side by a sliding piston that is connected to the spring. The piston travels within the cylinder without friction. Two stops are provided to limit the piston's movement and to define the minimum and maximum volumes occupied by the working body (figure 1). In the ultimate states of maximum compression and extension of the spring its potential energy is maximum. In the intermediate state where the spring is relaxed, its potential energy is zero, while the kinetic energy of the working body and the piston is not.

The Carnot heat engine is put into operation by introducing an amount of potential energy into the engine. We will call this energy the start-up energy. For example, when the gas has the minimum volume in contact with the heater, the spring is extended to maximum (figure 2*a*). In this case, the start-up energy has the form of the potential energy of the extended spring. The start-up energy can take other forms. For example, it can be added as a surplus pressure of the working body compared with the environment or as the kinetic energy of the working body and the piston.

The cycle starts when the spring is extended to its utmost. At this moment the gas occupies the smallest volume (point a in figure 2*a*) and gas pressure in the cylinder is equal to that of the external environment. The cylinder, which ideally has an infinite heat conductivity, is put in contact with the heater at $T = T_\mathrm{h}$. The spring starts compressing and moves the piston to the right such that the volume occupied by the working body increases at a constant temperature (the warmer isotherm of the Carnot cycle). The working body receives heat and, together with the spring, performs mechanical work on moving the piston. After the amount of received heat reaches Q_h, point b in figure 2*b*, the contact with the heater is mechanically disrupted. The gas further expands adiabatically until its temperature diminishes from T_h to T_c, point c in figure 2*c*. At this point, the working body is brought into contact with the cooler at $T = T_\mathrm{c}$.

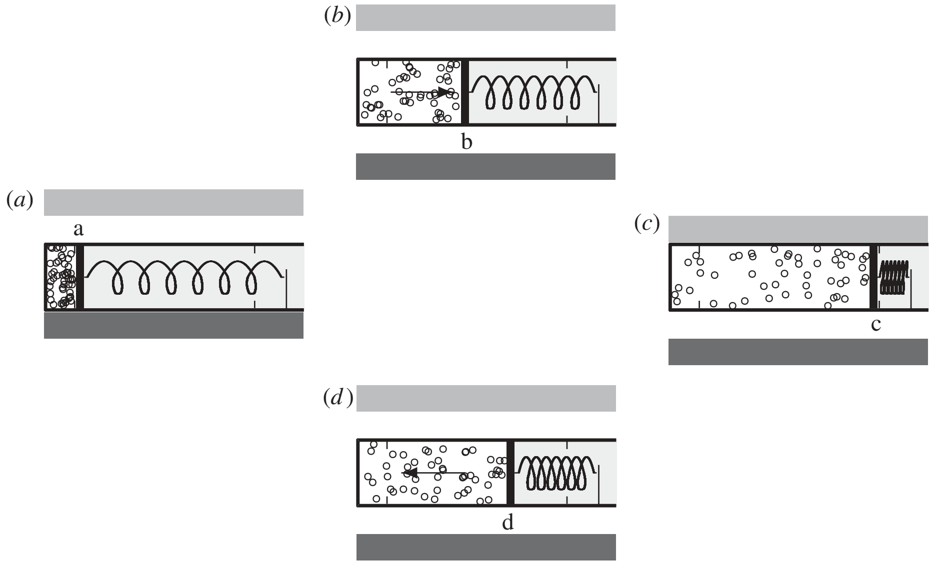

Figure 2. Carnot cycle. (*a*) Beginning of the warmer isotherm at $T = T_h$; the working body is brought into contact with the heater, the spring pulls the piston to the right and the gas expands. The initial pressure p of gas at point a coincides with that of the external environment. (*b*) End of the warmer isotherm, beginning of the first adiabate; the heater is detached from the engine, the gas expands adiabatically and its temperature drops from $T = T_h$ to $T = T_c$. (*c*) End of the first adiabate, beginning of the colder isotherm at $T = T_c$; the working body is brought into contact with the cooler, the spring extends and pushes the piston to the left. (*d*) End of the colder isotherm, beginning of the second adiabate; gas is compressed by the moving piston and the gas temperature increases from $T = T_c$ to $T = T_h$. Note that at the end of the cycle the piston has acquired kinetic energy equal to net work A performed by the working body in the cycle. To keep the engine stationary, this energy should be taken away from the engine at point a. Note that, in order for the associated air velocity to be sufficiently low so as not to disturb the thermodynamic equilibrium, the piston should be made sufficiently heavy.

The piston velocity at point c becomes zero; the spring starts extending and compresses the working body, which allows the latter to dispose heat to the cooler. At point d in figure 2*d*, the amount of disposed heat reaches Q_c, the cooler is detached from the cylinder and the gas continues to be compressed adiabatically. Its temperature rises back to T_h, point a in figure 2*a*. At this point, the Carnot cycle is completed. Work A performed by the gas has taken the form of the kinetic energy of the piston and can be used outside the engine. If this work is not taken away, it will continuously accumulate within the engine, increasing the piston velocity and kinetic energy with each cycle. As a result, the power of the engine (i.e. the number of cycles per unit time) will increase.

Two aspects need to be emphasized. First, the start-up energy is principally important for the heat engine to operate. If there is no spring, and the gas in the cylinder is in thermodynamic equilibrium with the heater and with the environment, the piston will remain immobile, the engine will not operate and no work will be produced. In the case of an infinite environment (figure 1*a*),

in which pressure p does not change during the cycle, the necessary amount of the start-up energy E ($\mathrm{J\,mol^{-1}}$) can be calculated as the difference between work A_p performed by the piston on the environment with constant pressure p and work A_w performed by the working gas with $p(v) \le p$ on the piston as the piston moves from point a to point c, $E = A_\mathrm{p} - A_\mathrm{w} = \int_\mathrm{a}^\mathrm{c}(p - p(v))\mathrm{d}v = p\Delta v - \int_\mathrm{a}^\mathrm{c} p(v)\mathrm{d}v \ge 0$, where v is molar volume, $pv = RT$ is the equation of state for the ideal gas and R is the universal gas constant. It is easy to see that, at $\Delta v/v_\mathrm{a} \sim 1$, $\Delta v \equiv v_\mathrm{b} - v_\mathrm{a}$, the start-up energy E should be of the order of $Q_\mathrm{h} = \int_\mathrm{a}^\mathrm{b} p(v)\mathrm{d}v = RT_\mathrm{h}\ln(1 + \Delta v/v_\mathrm{a})$. Indeed, we have $E \ge p\Delta v - RT_\mathrm{h}\ln(1 + \Delta v/v_\mathrm{a}) \approx p\Delta v - RT_\mathrm{h}(\Delta v/v_\mathrm{a}) + RT_\mathrm{h}(\Delta v/v_\mathrm{a})^2/2 \sim RT_\mathrm{h}/2$ at $\Delta v/v_\mathrm{a} \sim 1$ and $p = p_\mathrm{a}$. This is a conservative estimate that ignores work performed on the adiabate b–c where the working body continues to expand.

Second, if the heat conductivity of the heater and the cooler is sufficiently large, it strictly ensures constant temperature as the piston moves from point a to point b at $T = T_\mathrm{h}$ and from point c to point d at $T = T_\mathrm{c}$. Therefore, the amounts of heat Q_h and Q_c received and given away, respectively, by the working body are unambigously determined by the construction of the engine. At a given T_h, the value of Q_h is determined by the change of molar volume v from point a to point b, $Q_\mathrm{h} = \int_\mathrm{a}^\mathrm{b} p\,\mathrm{d}v \approx p\Delta v$ for small relative changes of gas pressure $p(v)$.

The first and second laws of thermodynamics for the Carnot cycle take the form

$$Q_\mathrm{h} - A = Q_\mathrm{c} \tag{2.1}$$

and

$$\frac{Q_\mathrm{h}}{T_\mathrm{h}} = \frac{Q_\mathrm{c}}{T_\mathrm{c}}. \tag{2.2}$$

The five magnitudes entering equations (2.1) and (2.2) leave three out of the five variables independent, e.g. Q_h, T_h and T_c. Equations (2.1) and (2.2) can be written as

$$A = \varepsilon Q_\mathrm{h}, \quad Q_\mathrm{c} = (1 - \varepsilon)\,Q_\mathrm{h} \quad \text{and} \quad \varepsilon \equiv \frac{T_\mathrm{h} - T_\mathrm{c}}{T_\mathrm{h}}. \tag{2.3}$$

Here two variables are independent, Q_h and ε that depend on T_h and T_c. From equation (2.3), we obtain by replacing the independent variable Q_h by Q_c

$$A = \frac{\varepsilon}{1 - \varepsilon}Q_\mathrm{c} \quad \text{or} \quad A = \varepsilon(Q_\mathrm{c} + A), \quad \frac{\varepsilon}{1 - \varepsilon} = \frac{T_\mathrm{h} - T_\mathrm{c}}{T_\mathrm{c}}. \tag{2.4}$$

It is noteworthy that these relationships coincide with those for the dissipative heat engine if one replaces Q_c by a formally introduced variable

$$Q_\mathrm{hd} \equiv Q_\mathrm{c}, \tag{2.5}$$

where Q_hd refers to the external heat input in the dissipative heat engine. Note that at $T_\mathrm{c} \to 0$ in equation (2.5) work A remains finite in view of equation (2.2): at constant Q_h and T_h, the decrease in T_c must be accompanied by a proportional decrease in Q_c.

3. Physical meaning of the mathematical relationships of the dissipative heat engine

Thus, the mathematics of the dissipative heat engine formally rewrite the first and second laws of thermodynamics for the Carnot heat engine, equations (2.1) and (2.2), by introducing a new variable $Q_{\mathrm{hd}} \equiv Q_{\mathrm{c}}$. As shown in the previous section, work A can be produced by the Carnot heat engine after it is supplied with the start-up energy and the working body receives heat Q_{h} from the heater. After work is performed in the first cycle, it can, in principle, be dissipated into heat at $T = T_{\mathrm{h}}$ and introduced into the working body at the first stage of the second cycle. We note that, although principally possible, such a procedure is technically difficult, as it demands a resonance between the dynamics of the piston movement and that of the dissipation process. Characteristic times of the piston movement and the dissipation process are dictated by different physical laws and generally do not coincide. Resonance synchronization of independent physical processes is in the general case impossible. (For example, it would need to be proved that all kinetic energy originating within a hurricane dissipates within the hurricane and is not transported away in the upper atmosphere to dissipate to heat outside the hurricane area, i.e. that this internal dissipation is synchronized with the supposed heat input from the ocean (the heater). Without such a proof, the application of the main equation of the dissipative heat engine, $A_{\mathrm{d}} = \varepsilon(Q_{\mathrm{hd}} + A_{\mathrm{d}})$ (Emanuel 2003), is not valid.)

When work A dissipates to heat within the working body, the latter warms. The Carnot heat engine is originally constructed such that, while the piston moves from point a to point b (figure 2), the working body receives heat Q_{h}. This is possible due to the fact that, when the piston moves and the volume of the gas increases, the gas becomes a little colder than the heater, enabling the necessary heat to flow from the heater to the engine. If the working body now has a source of heat inside, the extension of the working gas due to piston movement does not sufficiently decrease the gas temperature to ensure the same flux Q_{h} from the heater. As prescribed by the first law of thermodynamics and the ideal gas equation, gas that isothermally expands by a preset amount (from point a to point b in figure 2) receives a fixed amount of heat $Q_{\mathrm{h}} = \int_{\mathrm{a}}^{\mathrm{b}} p\,\mathrm{d}v$. If some part of this heat $Q_A = A$ is delivered to the working body as the product of dissipation of work A, then the amount of external heat Q_{hd} received by the engine from the heater will *decrease* compared with the Carnot heat engine to $Q_{\mathrm{hd}} = Q_{\mathrm{h}} - A$. Since the amount of heat received by the working body remains unchanged, as dictated by the geometry of the working cylinder (figure 1), the amount of work A performed by the engine is also constant and cannot be increased by dissipating work performed by the engine in the previous cycles.

4. The impossibility of the dissipative heat engine

Thus, it is the central idea that 'the fraction of mechanical energy dissipated ... increases the heat input to the convective heat engine' (Rennó & Ingersoll 1996, p. 579) is the main physical inconsistency in the concept of the dissipative heat engine, which brings the concept into conflict with the laws of thermodynamics.

Let $Q_{\mathrm{hd}} = Q_{\mathrm{h}}$ be the total amount of heat received by the dissipative heat engine in the first cycle (no work previously produced). Work $A_1 = A$ produced during this cycle is determined by Carnot equation (2.3). In the subsequent cycles, heat formed due to the dissipation of work produced in the preceding cycle is added to the fixed amount of external heat Q_{hd} received from the heater

$$A_{n+1} = \varepsilon(Q_{\mathrm{hd}} + A_n) \quad \text{and} \quad A_1 = A = \varepsilon Q_{\mathrm{hd}}, \tag{4.1}$$

where $n \geq 1$ is the number of the current cycle. We have from equation (4.1)

$$A_n = \varepsilon(Q_{\mathrm{h}})_n, \quad (Q_{\mathrm{h}})_n \equiv Q_{\mathrm{hd}}(1 + \varepsilon + \varepsilon^2 + \cdots + \varepsilon^{n-1}) \tag{4.2}$$

and

$$(Q_{\mathrm{cd}})_n = (Q_{\mathrm{h}})_n - A_n = Q_{\mathrm{hd}}(1 - \varepsilon^n). \tag{4.3}$$

Here, A_n is work produced in the nth cycle; $(Q_{\mathrm{h}})_n$ is the total heat received by the working body (external heat plus the dissipative heat from work performed in the previous cycles)—note that, according to the dissipative heat engine concept, $(Q_{\mathrm{h}})_n$ grows with n following the increase in internally dissipated work; $(Q_{\mathrm{cd}})_n$ is the amount of heat given away to the cooler, it decreases with growing n at $\varepsilon < 1$. As is easy to see, at $n \to \infty$ work, $A_{\mathrm{d}} = A_\infty$ is formally determined by the equation of the dissipative heat engine

$$A_{\mathrm{d}} = \varepsilon(Q_{\mathrm{hd}} + A_{\mathrm{d}}) \quad \text{and} \quad Q_{\mathrm{cd}} = (1 - \varepsilon)(Q_{\mathrm{hd}} + A_{\mathrm{d}}) = Q_{\mathrm{hd}} \tag{4.4}$$

and

$$A_{\mathrm{d}} = \frac{\varepsilon}{1 - \varepsilon} Q_{\mathrm{hd}} = \frac{T_{\mathrm{h}} - T_{\mathrm{c}}}{T_{\mathrm{c}}} Q_{\mathrm{hd}}. \tag{4.5}$$

A similar derivation can be performed for a general case when not all but some part $\gamma \leq 1$ of work A_n produced in the cycle is dissipated at the warmer isotherm, $A_{n+1} = \varepsilon(Q_{\mathrm{hd}} + \gamma A_n)$, cf. equation (4.1), while the rest of the produced work, $(1 - \gamma)A_n$, is dissipated at the colder isotherm with the resulting heat dispatched to the cooler (Rennó & Ingersoll 1996). We then obtain

$$A_{\mathrm{d}} = \frac{\varepsilon}{1 - \gamma\varepsilon} Q_{\mathrm{hd}} = \frac{T_{\mathrm{h}} - T_{\mathrm{c}}}{(1 - \gamma)T_{\mathrm{h}} + \gamma T_{\mathrm{c}}} Q_{\mathrm{hd}}. \tag{4.6}$$

In the case of zero dissipation at the warmer isotherm, $\gamma = 0$ and $Q_{\mathrm{hd}} = Q_{\mathrm{h}}$, equation (4.6) coincides with equation (2.3) for work A of the Carnot cycle.

Equations (4.5) and (4.6) say that work A_{d} performed by the dissipative heat engine increases infinitely at fixed Q_{hd} with $T_{\mathrm{c}} \to 0$. Thus, the dissipative heat engine represents an engine that recirculates heat to work and back at a potentially infinite power; it produces work A_{d} greater than it would in the absence of dissipation, i.e. greater than the work A of the Carnot heat engine that operates with the same external heat input $Q_{\mathrm{hd}} = Q_{\mathrm{h}}$, cf. equations (4.5) and (2.3). An infinite power of this recirculation can be achieved by simply decreasing temperature T_{c} of the cooler. Such an engine violates both the first and the second laws of thermodynamics and cannot exist.

Indeed, since for a given heat engine $Q_{\mathrm{h}} = Q_{\mathrm{hd}} + A_{\mathrm{d}} = \text{const.}$, work A_n produced in each cycle is also constant, $A_n = \varepsilon Q_{\mathrm{h}} = A$. This means that no accumulation of mechanical work beyond A within the engine is possible in any cycle. Thus,

the property $A_d > A$ of the dissipative heat engine would violate the energy conservation law (the first law of thermodynamics), because the difference $A_d - A > 0$ remains unaccounted for by the energy balance. In order to increase the heat input to the engine, it is necessary to reconstruct it. At constant T_h and T_c, such a reconstruction would imply, first, an increase in the start-up energy and, second, an increase in the linear size of the engine to allow for a greater expansion of the working body at the warmer isotherm (e.g. Leff 1987). Internal dissipation of produced work within the engine does not lead to an increase in the heat input.

In the Carnot cycle, all heat Q_h received by the working body on the warmer isotherm is converted to work with an entropy increase $S_h = Q_h/T_h = (\int_a^b p\,dv)/T_h$ that corresponds to the isothermal gas expansion as dictated by the construction of the engine. Thus, the same condition $Q_{hd} + A_d > Q_h$ of the dissipative heat engine would mean an additional dissipation of work A_d to heat and its regeneration back to work A_d from heat at one and the same temperature, which is prohibited by the second law of thermodynamics $dS \geq dQ/T$. Indeed, for the warmer isotherm, we would then have instead

$$\Delta S = \frac{\int_a^b p\,dv}{T_h} \leq \frac{\Delta Q}{T_h} = \frac{Q_{hd} + A_d}{T_h}. \tag{4.7}$$

5. Conclusions and discussion

(a) *Internal dissipation cannot increase the work output of a heat engine*

Equations (4.5) and (4.6) for work A_d of a heat engine where the work produced is dissipated within the engine are incorrect. The problem consists in the fact that with increasing dissipation rate A_n the external heat input in equations (4.1) and (4.4) does not remain constant, but decreases as $Q_{hd} = Q_h - A_n$, while total heat input Q_h to the working body remains constant as prescribed by the construction of the engine in question. Putting $Q_{hd} = Q_h - A_d$ into equation (4.5) gives $Q_{hd} = (1-\varepsilon)Q_h = (T_c/T_h)Q_h$. Therefore, at $T_c \to 0$, we have $Q_{hd} \to 0$ (no heat input from the heater) and work $A_d = A = \varepsilon Q_h = \text{const.}$ remains limited and equal to that of a Carnot heat engine where no dissipation takes place. We summarize that dissipation of work within a heat engine *cannot increase the work produced by the engine*.

In the general case, work $A = A_d = \text{const.}$ produced by the engine does not depend on the proportion γ of mechanical work dissipated at the warmer isotherm. Equation (4.6) is physically misleading, as it hides the dependence of Q_{hd} on γ and Q_h, $Q_{hd} = Q_h - \gamma A_d$, which, when entered into the equation, yields $A_d = \varepsilon Q_h = \text{const.}(\gamma)$.

It is important to note that, as far as $Q_h = Q_{hd} + A_d = \text{const.}$, the entropy balance equation for the dissipative heat engine $S_d = Q_{hd}/T_h + A_d/T_h - Q_c/T_c = Q_h/T_h - Q_c/T_c = 0$ is mathematically identical to the entropy balance equation of the Carnot heat engine, where one term Q_h/T_h is formally divided into two, $Q_h/T_h \equiv Q_{hd}/T_h + A_d/T_h$. The problem with the dissipative heat engine is physical, not mathematical. It could not have been revealed from a formal consideration of the engine's entropy budget without considering the essential physical peculiarities of what a heat engine is.

(*b*) *Related problems in applying the heat engine concept to the atmosphere*

In the meantime, the available theoretical considerations of atmospheric circulation on the basis of a thermodynamic cycle such as the Carnot cycle (Rennó & Ingersoll 1996; Rennó 1997, 2001; Pauluis *et al.* 2000; Pauluis & Held 2002; Lucarini 2009) concentrate on evaluating the energy and entropy budgets but refrain from addressing the dynamic and structural aspects of the heat engine as a physical entity characterized by inherent spatial and temporal scales. Namely, this has caused the dissipative heat engine inconsistency discussed in this paper, but the conceptual problem appears to be wider and warrants further investigations.

In particular, the necessity for an auxilliary dynamic system that would possess the start-up energy E to expand and contract the working body of the atmospheric heat engine, the air, is not taken into account. The classical Carnot heat engine is based on equilibrium thermodynamics, which prescribes that all the non-equilibrium processes of the cycle such as heat transfer from the heater to the working body occur at an infinitely small rate. Therefore, the power (work performed per unit time) of the ideal Carnot heat engine is infinitely small.

Efficiency $\varepsilon_{\mathrm{mp}}$ corresponding to maximum power output of real heat engines is not equal to Carnot efficiency, but approaches $\varepsilon_{\mathrm{mp}} = 1 - \sqrt{T_{\mathrm{c}}/T_{\mathrm{h}}}$ as first pointed out by Curzon & Ahlborn (1975). This result was confirmed by rigorous theoretical investigations recently performed in various domains of science (Leff 1987; Rebhan 2002; Van den Broeck 2005; Feidt *et al.* 2007; Jiménez de Cisneros & Calvo Hernández 2008; Izumida & Okuda 2009). This result was, however, neglected in the formulation of the maximum potential intensity model for hurricanes based on the heat engine concept (e.g. Emanuel 2003). This points to a breach between the treatments of the heat engine concept in modern climatology versus theoretical physics.

A working body in thermodynamic equilibrium with an infinite heat source (ocean) cannot spontaneously start expanding and receiving heat at a finite rate; this is thermodynamically prohibited. As is known from the studies of finite-time thermodynamics of heat engines that have been recently gaining momentum (Rebhan 2002; Van den Broeck 2005; Feidt *et al.* 2007; Izumida & Okuda 2009), the time scale for operation of heat engines producing finite power is set externally by the auxilliary system (e.g. the mechanism that moves the piston at a given velocity). As the piston is externally moved, the temperature of the gas is lowered and there appears to be a temperature difference enabling a heat flow from the hot reservoir to the working body. This allows for a finite rate of heat flow to the engine. This temperature difference grows with increasing piston velocity (Izumida & Okuda 2009), while the engine efficiency decreases.

It is not that some parts of the engine move because the engine receives heat from the heater, but, conversely, the engine can receive heat only because some parts of the engine are moved by the auxilliary mechanical system. In the atmosphere, no independent physical mechanism has ever been identified that would compress and decompress the air and transport it between the cold and hot reservoirs (e.g. between the surface and upper troposphere, as in the hurricane model of Emanuel (2003)) at a finite velocity in a manner similar to how it is done by the piston-moving mechanism in real heat engines. Unless such a mechanism is described, the horizontal drop in air pressure observed in hurricanes cannot be explained as the outcome of a Carnot heat engine operating in the atmosphere. The need for an independent specification of a horizontal pressure

gradient was identified as a fundamental problem of the thermodynamic approach to hurricane formation (Smith *et al.* 2008). In parallel, it was recently proposed that the nature of atmospheric circulation is dynamic, not thermodynamic, and relates to the release of potential energy during condensation of water vapor (Makarieva & Gorshkov 2007, 2009*a*,*b*). In summary, the problem of applicability of the heat engine concept to the atmosphere, to which the present analysis has aimed to contribute, appears to justify the broader attention of scientists from different fields.

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