Hurricane's Maximum Potential Intensity and Surface Heat Fluxes

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Abstract. The concept of Maximum Potential Intensity (PI) relates the maximum velocity of tropical storms to environmental parameters. Since its original formulation by Emanuel (1986), two major modifications were made resulting in a considerable range of predicted PI values. First, dissipative heating was suggested to increase maximum velocity by up to 20%. Second, the power expended to lift water (the gravitational power of precipitation) was recently suggested to reduce maximum velocities by 10-30%. Here we re-derive the PI concept separating its dynamic and thermodynamic assumptions. First, we show that there is no ambiguity from dissipative heating: irrespective of whether thermal dissipation occurs or not, the PI concept uniquely relates maximum velocity to the latent heat flux (not the total oceanic heat flux as in the original formulation). Our revised velocity estimate is independent of sensible heat flux. Second, we clarify that accounting for the gravitational power of precipitation has little impact on PI. Third, we demonstrate that a key feature of the PI concept is that the negative work of the pressure gradient in the upper atmosphere consumes all the kinetic energy generated in the boundary layer. This dynamic constraint is independent of thermodynamic assumptions such as isothermal heat input from the ocean, and thus can apply to diverse circulation patterns. Finally, we show that the maximum kinetic energy per unit volume in the PI concept is approximately equal to the partial pressure of water vapor at the surface.

1 Introduction

Predicting hurricane intensity is a challenge. In the modern literature *intensity* can refer either to the maximum pressure fall (e.g., Malkus and Riehl, 1960; Emanuel, 1988; Holland, 1997; Hart et al., 2007) or to the maximum sustained velocity within a storm (e.g., Emanuel, 1986; Camp and Montgomery, 2001; Kowaleski and Evans, 2016).

Early theoretical studies focused on pressure. Given that the hurricane is warmer than the ambient environment the idea was to retrieve the surface pressure deficit from this extra warmth assuming the existence of an unperturbed atmospheric top where pressures in the hurricane and the environment coincide. Since air pressure drops with altitude more slowly when the atmosphere is warm than when it is cold, to arrive at equal pressures at the top of the troposphere one must start from a lower surface pressure in the warmer column. The height of the unperturbed top and the extra warmth of the hurricane compared to its environment uniquely determined the surface pressure deficit in the storm. It has been known that this pressure deficit is well correlated with maximum velocity (Holland, 1980; Willoughby et al., 2006; Holland, 2008; Kossin, 2015; Chavas et al., 2017). However, it was challenging to describe this correlation from theory rather than observational fitting and thus retrieve a velocity value from the predicted pressure deficit.

Emanuel (1986) noted that the wind generating work associated with surface pressure deficit is constrained not only by the first law of thermodynamics, but also by the Bernoulli equation that derives from the equations of motion and continuity. Combining these and assuming that both the generation of kinetic energy and its dissipation (proportional to the cube of velocity) occur within the boundary layer, he linked work to power to obtain explicit formulae for calculating maximum hurricane velocity from environmental parameters. This upper limit became widely used (see recent discussions by Garner (2015), Kieu and Moon (2016) and Kowaleski and Evans (2016)).

The key equation of Emanuel's PI concept (E-PI hereafter) relates local surface fluxes of turbulent dissipation D and oceanic energy flux J (W m^{-2}) in the region of maximum winds:

$$D = \varepsilon_C J, \tag{1}$$

where

$$J = \rho C_k V_{\max}(k_s^* - k) = \rho C_k V_{\max}(c_p \Delta T + L\Delta q), \tag{2}$$

$$D = \rho C_D V_{\max}^3, \tag{3}$$

such that

$$V_{\max}^2 = \varepsilon_C \frac{C_k}{C_D}(k_s^* - k). \tag{4}$$

Here $\varepsilon_C = (T_a - T_o)/T_a$ is the efficiency of a Carnot cycle operating on a temperature difference between the ambient temperature T_a in the boundary layer and temperature T_o of air outflow in the upper atmosphere; ρ is air density, $C_k \approx C_D \sim 10^{-3}$ are surface exchange coefficients for enthalpy and momentum, respectively; k_s^* (J kg^{-1}) is saturated enthalpy of air at surface temperature and $k \approx c_p T + Lq$ is the actual enthalpy of air in the boundary layer; $\Delta q \equiv q_s^* - q_b$ is the difference between saturated water vapor mixing ratio q_s^* at the oceanic surface and the actual mixing ratio q_b in the boundary layer; $\Delta T \equiv T_s - T_b$ is, likewise, the difference between the temperature of the oceanic surface and the temperature of the adjacent air in the boundary layer, $V_{\max}$ is tangential velocity, which at the radius of maximum winds approximates total air velocity (the vertical and radial velocities are relatively small).

Equation (1) is remarkable as it relates *local* power of turbulent dissipation in the boundary layer to *local* heat flux from the ocean via efficiency ε_C, despite the latter is not a local characteristics but applies to a Carnot cycle as a whole. As we discuss below, this local property follows from the dynamic assumptions of E-PI, in particular, from the assumption of gradient wind balance above the boundary layer. Smith et al. (2008) argued that E-PI should underestimate maximum velocity as it implicitly applies the gradient wind balance to the boundary layer where this assumption does not hold. Tang and Emanuel (2012) agreed that a correction to $V_{\max}$ of about 10-15% accounting for the boundary layer dynamics is justified, which is approximately what is found in numerical simulations (Wang and Xu, 2010; Frisius et al., 2013).

Apart from the gradient wind balance discussions, the PI concept advanced through several modifications aimed to remedy the limitations of the original formulation (Emanuel, 1988, 1991, 1995, 1997; Tang and Emanuel, 2012). A major modification was proposed by Bister and Emanuel (1998) who suggested that Eq. (1) should be replaced by $D = \varepsilon_C(J+D)$. They interpreted this as a situation when turbulent kinetic energy locally dissipates to heat and this heat is added to the thermodynamic cycle along with the oceanic heat source J. This modification of (1) leads to replacement of ε_C by $\varepsilon_C/(1-\varepsilon_C)$ in (4), which increases $V_{\max}$ by about 20% for a typical hurricane value of $\varepsilon_C \sim 0.3$ (Emanuel, 1986; DeMaria and Kaplan, 1994). However, it is not obvious whether turbulent dissipation within the hurricane proceeds down to the thermal level or the ultimate products of this dissipation are small-scale eddies that are exported from the hurricane without contributing to its heat balance. The criteria by which to decide whether thermal dissipation occurs or not are unclear; the dissipative heating can be insignificant (Kieu, 2015). To account for this uncertainty, it became common to estimate $V_{\max}$ both with and without dissipative heating (e.g., Montgomery et al., 2006; Sabuwala et al., 2015). This created a wide range of predicted values. E.g. if $V_{\max} \sim 70$ m s^{-1} is a Category 5 hurricane, a 20% reduction in $V_{\max}$ makes it a Category 3.

Recently, Sabuwala et al. (2015) proposed another significant modification to E-PI suggesting that Eq. (1) should become $D + W_P = \varepsilon_C(J + D)$, to account for the power W_P needed to lift precipitating water. Sabuwala et al. (2015) estimated W_P from the observed rainfall in the region of maximum wind. They concluded that accounting for W_P can reduce $V_{\max}$ by as much as 30%. Since theoretical estimates of hurricane W_P appear unavailable (but see Makarieva and Gorshkov, 2011), this proposition created further uncertainty in E-PI $V_{\max}$ values.

Understanding what determines hurricane intensity is crucial for improved predictions. Here we re-derive E-PI separating the dynamic and thermodynamic assumptions of the concept and, in the result, demonstrate that it produces a considerably more unambigous $V_{\max}$ estimate than it would currently appear. We show that the correct version of Eq. (1) consistent with E-PI assumptions is $D = \varepsilon_C(D + J_L)$, where $J_L \leq J$ is the latent heat flux from the ocean. As a consequence, E-PI $V_{\max}$ is formally independent of the presence or absence of dissipative heating as well as of the sensible heat flux. Furthermore, we show that, contrary to the suggestion of Sabuwala et al. (2015), the gravitational power of precipitation W_P makes only a minor impact on hurricane intensity, although this impact grows with decreasing ε_C.

Our analysis demonstrates that the dynamic and thermodynamic assumptions of E-PI are logically independent. The key dynamic assumption of E-PI constrains cumulative kinetic energy generation in the upper and lower atmosphere and can be applied to diverse circulation patterns. Finally, we perform a scale analysis of key parameters determining $V_{\max}$ in E-PI and show that, in agreement with our previous suggestion (Makarieva et al., 2017a), numerically E-PI implies $\rho V_{\max}^2/2 \sim p_{vs}$, where p_{vs} is partial pressure of water vapor in the surface air. We outline ideas requiring further investigation.

2 Emanuel's Potential Intensity

2.1 Dynamics

Emanuel (1986) assumed, first, that the air is in hydrostatic equilibrium,

$$\alpha \frac{\partial p}{\partial z} = -g, \tag{5}$$

where $\alpha \equiv 1/\rho$, which means that for any closed contour in the atmosphere

$$\oint \alpha dp = \oint \alpha \frac{\partial p}{\partial r} dr + \oint \alpha \frac{\partial p}{\partial z} dz = \oint \alpha \frac{\partial p}{\partial r} dr. \tag{6}$$

Second, the air is in gradient wind balance above the boundary layer:

$$\alpha \frac{\partial p}{\partial r} = \frac{V^2}{r} + fV, \quad z \geq h_b. \tag{7}$$

Here V is tangential velocity; under approximations (5) and (7) it is equal to total velocity (the radial and vertical velocities are negligible). The Coriolis parameter $f \equiv 2\Omega \sin\varphi$ is assumed constant (φ is latitude, Ω is the angular velocity of Earth's rotation).

Using (7) for any path $x-y$ along which the angular momentum

$$M \equiv Vr + \frac{fr^2}{2} \tag{8}$$

is constant, one obtains

$$-\int_x^y \alpha \frac{\partial p}{\partial r} dr = -\int_x^y \left(\frac{V^2}{r} + fV \right) dr = \left. \frac{V^2}{2} \right|_x^y. \tag{9}$$

Now consider a closed contour $a-c-o-o'-a$ (Fig. 1), where at $a-c$ and $o-o'$ we have, respectively, $z = h_b$ and $r = r_o$, while at $c-o$ and $o'-a$ the angular momentum is constant. Using (6) and applying (9) to the momentum-conserving paths $c-o$ and $o'-a$ we have

$$-\oint \alpha dp = -\int_a^c \alpha \frac{\partial p}{\partial r} dr - \frac{1}{2} \left(V_c^2 - V_a^2 + V_{o'}^2 - V_o^2 \right). \tag{10}$$

Note that (10) is valid independent of whether the gradient wind balance applies to the horizontal path $a-c$.

To clarify the physics of this result, let us imagine that the air actually moves along the considered contour, i.e. that $a-c-o-o'-a$ is a real streamline such that the Bernoulli equation applies:

$$d\left(\frac{v^2}{2}\right) + \alpha dp + g dz - \mathbf{f} \cdot d\mathbf{l} = 0. \tag{11}$$

Here v is air velocity ($v \approx V$ under our assumptions), $\mathbf{f}$ is the friction force per unit air mass and $d\mathbf{l} = \mathbf{v} dt$, see Emanuel (1986, Eq. 64) and Emanuel (1988, Eq. C1).

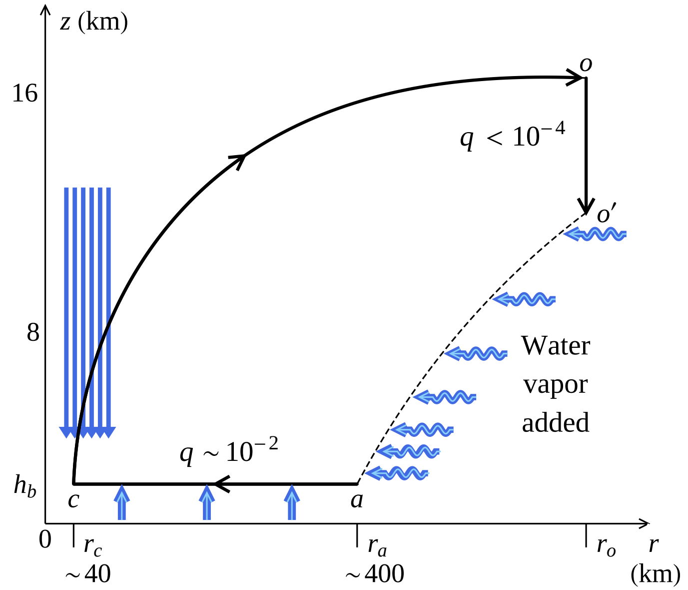

Figure 1. Key features of a hurricane relevant for E-PI. The solid curve $a - c - o - o'$ is the actual streamline of air entering the hurricane at point a and leaving it at point o'; dashed curve $o' - a$ is the hypothetical path closing the thermodynamic cycle $a - c - o - o' - a$ with two isotherms, $a - c$ and $o - o'$, and two adiabats, $c - o$ and $o' - a$. As the air rises from c to o, water vapor condenses and precipitates, water vapor mixing ratio $q \equiv \rho_v/\rho_d$, where ρ_v is water vapor density, ρ_d is dry air density, declines by over two orders of magnitude. A major part of this lost water re-appears in the cycle along the hypothetical $o' - a$ adiabat (as shown by waved arrows); in real hurricanes this imported moisture derives from evaporation outside the storm and is picked up as the storm moves through the atmosphere. The remaining part of moisture lost as rainfall is provided by evaporation from the sea surface (straight upward arrows). Straight downward arrows indicate the rainfall maximum that occurs in the vicinity of the radius of maximum wind $r = r_c$; r_a corresponding to point a is an external radius of the storm estimated to be approximately an order of magnitude larger than r_c (Emanuel, 1995, Table 1, note that r_a is denoted there as r_0 and r_c as r_m); $z_a = z_c = h_b$ is the height of the boundary layer.

Assuming that air density is constant along $a - c$ and using (11) we find

$$A^+ \equiv -\int_a^c \alpha dp = -\frac{\delta p}{\rho} = \frac{1}{2}\left(V_c^2 - V_a^2\right) - \int_a^c \mathbf{f} \cdot d\mathbf{r}, \tag{12}$$

where $\delta p \equiv p_c - p_a$. This relationship shows that the work of the pressure gradient on the horizontal part of the streamline goes to increase kinetic energy and to generate some turbulence. (As the air ascends in the region of maximum winds, V grows with decreasing r such that $V_c > V_a$.) Notably, the turbulence term $-\int_a^c \mathbf{f} \cdot d\mathbf{r} > 0$ is positive, since according to observations the first term in the right-hand side of (12) – the kinetic energy increment – is smaller than work of the pressure gradient $-\delta p/\rho$ (see, e.g., Knaff and Zehr, 2007, Table 1).

On the other hand, from (10) we notice that the work A^- of the pressure gradient on the remaining part of the streamline $c-o-o'-a$, i.e. the total work performed by the pressure gradient above the boundary layer, is negative:

$$A^- \equiv -\int\limits_{c-o-o'-a} \alpha dp = -\int\limits_{c-o-o'-a} \alpha \frac{\partial p}{\partial r} dr = -\frac{1}{2}\left(V_c^2 - V_a^2 + V_{o'}^2 - V_o^2\right). \tag{13}$$

(Note that since $M_{o'} > M_o$ and $r_{o'} = r_o$, we have $V_{o'}^2 > V_o^2$, i.e. not a loss but an increment of kinetic energy occurs along $o-o'$.) Such areas where the generation of kinetic energy by the radial pressure gradient is negative are indeed noticeable at least in modelled hurricanes – see, for example, Fig. 42 (panel KB) of Kurihara (1975) and Fig. 5a,b of Smith et al. (2018).

Let us temporarily neglect the last two terms in (13). Then from a joint consideration of (12) and (13) we can conclude that in E-PI all kinetic energy generated by the radial pressure gradient in the lower atmosphere is then spent in the upper atmosphere as the air moves *against* the radial pressure gradient force. This dynamic constraint is a distinct property of E-PI.

In earlier works, such as that of Malkus and Riehl (1960), it was assumed, by postulating the existence of an isobaric height, that work in the upper atmosphere is not negative but zero. Indeed, if the air moves along the isobaric height, $\partial p/\partial r = 0$, the work of the pressure gradient is zero and $A^- = 0$. If $\varepsilon_C Q = A^+ + A^-$, where Q is heat input to the cycle, then for one and the same $\varepsilon_C Q$ a negative $A^- < 0$ results in a higher $A^+ \equiv -\delta p/\rho = \varepsilon_C Q - A^-$ than does $A^- = 0$. This difference in basic dynamic assumptions, $A^- \approx -(V_c^2 - V_a^2)/2$ in E-PI versus $A^- = 0$ of Malkus and Riehl (1960), explains why the proportionality coefficient between $-\delta p$ (i.e. A^+) and the increment of the (saturated) equivalent potential temperature (which determines Q) is substantially higher in E-PI than in the derivations of Malkus and Riehl (1960) (cf. equation (22) and the following unnumbered equation on p. 589 of Emanuel, 1986).

Let us now discuss the increment of kinetic energy at $o-o'$. Applying the Bernoulli equation (11) together with the hydrostatic equilibrium (5) we find that this increment in the kinetic energy of the large-scale flow must be generated by turbulence:

$$\frac{1}{2}\left(V_{o'}^2 - V_o^2\right) - \int\limits_{o}^{o'} \mathbf{f}\cdot d\mathbf{l} = 0. \tag{14}$$

Indeed, in contrast to what happens on $a-c$, where $-\int_a^c \mathbf{f}\cdot d\mathbf{l} > 0$, on $o-o'$ the turbulence term is negative, $-\int_o^{o'} \mathbf{f}\cdot d\mathbf{l} < 0$, and therefore does not represent dissipation (which is positive definite). If we want the cycle to remain energetically closed, then we must assume that this energy increment arises as part of the turbulent energy generated on $a-c$ and is transported to $o-o'$, where it is somehow converted back to the kinetic energy of the main flow. However, as noted by Smith et al. (2014), a mechanism that would provide an increment of angular momentum, and hence a kinetic energy increment, along $o-o'$ does not appear to exist. Makarieva et al. (2017a) indicated that extra angular momentum can arise in the upper atmosphere as a real steady-state hurricane is an open system that moves through the atmosphere and can import angular momentum as it does air and water vapor. But in this case $o-o'$ should not be considered as an actual streamline and the Bernoulli equation should not be applied. We return to this issue in the next section.

Importantly, so far in Eqs. (5)-(14) we have not made any assumptions about the thermodynamics of the considered cycle. Thus the key dynamic constraint, $A^- \approx -(V_c^2 - V_a^2)/2$, can be applied to circulation patterns not conforming to the specific thermodynamic assumptions of E-PI (e.g. about the isothermal heat input from the ocean, to be discussed next) and, under different assumptions, can yield different results.

2.2 Thermodynamics

Emanuel (1986) assumed that $a-c$ with $T_a = T_c$ and $o-o'$ with $T_o = T_{o'}$ are isotherms, while the momentum-conserving paths $c-o$ and $o'-a$ are reversible adiabates. In this case the cycle achieves Carnot efficiency, $\varepsilon = \varepsilon_C = (T_a - T_o)/T_a$.

Work performed in the Carnot cycle per unit mass of dry air is

$$-\oint \alpha_d dp = \varepsilon_C \int_a^c \delta Q_d = \varepsilon_C \int_a^c (-\alpha_d dp + L dq), \tag{15}$$

where p is pressure, $\alpha_d \equiv 1/\rho_d$ is the specific volume of dry air, ρ_d is dry air density, δQ_d is heat received per unit dry air mass, ρ_v is water vapor density, $q \equiv \rho_v/\rho_d$, L is the latent heat of vaporization. In the second equality of (15) we have used the first law of thermodynamics to describe the isothermal heat input. Equation (15) indicates that once a moist air parcel containing m_d kg of dry air (plus some moisture) has completed the cycle, total work (in Joules) it has performed is given by (15) multiplied by m_d. Since, unlike the dry air mass m_d, the water vapor mass m_v may change during the cycle, the integral equation (15) cannot be written per unit wet air mass $m = m_d + m_v$.

Using the relationship

$$\alpha_d = \alpha(1+q) \tag{16}$$

and combining (10) and (15) we find

$$\varepsilon_C \int_a^c \left(-\alpha_d \frac{\partial p}{\partial r} + L\frac{\partial q}{\partial r}\right) dr = -\int_a^c \alpha \frac{\partial p}{\partial r} dr - \frac{1}{2}\left(V_c^2 - V_a^2 + V_{o'}^2 - V_o^2\right) - \oint \alpha q dp. \tag{17}$$

This relationship (17) is very similar to Equation (18) of Emanuel (1986), which underlies PI estimates (see Appendix A for calculation details). The difference arises because Emanuel (1986) did not discriminate between α and α_d and thus neglected the last term in (17).

Using (5), (6) and (9) we can express this last term as follows (see Appendix B for calculation details):

$$-\oint \alpha q dp = -\int_a^c q\left(\alpha \frac{\partial p}{\partial r} + \frac{1}{2}\frac{\partial V^2}{\partial r}\right) dr - \frac{1}{2}\left(q_{o'} V_{o'}^2 - q_o V_o^2\right) - \oint \frac{V^2}{2}\frac{\partial q}{\partial r} dr - \oint gz \frac{\partial q}{\partial z} dz. \tag{18}$$

Since (17) and (18) are valid for any points a and c on the isotherm $z = h_b$, we can consider a narrow contour[1] with $r_a \to r_c$ and $z_{o'} \to z_o$ (Fig. 1), such that Eq. (17), with an account of (18) and after some re-arrangement, becomes

$$\varepsilon_C \left(-\alpha \frac{\partial p}{\partial r} + \frac{L}{1+q}\frac{\partial q}{\partial r}\right) = -\alpha \frac{\partial p}{\partial r} - \frac{1}{2}\frac{\partial V^2}{\partial r} - \frac{1+q_o}{1+q}\frac{1}{2}\frac{\partial V_o^2}{\partial r} + \left(gH_P - \frac{V^2 - \overline{V^2}}{2}\right)\frac{1}{1+q}\frac{\partial q}{\partial r}, \tag{19}$$

[1]This procedure was suggested by Dr. Kerry Emanuel (pers. comm.).

$$H_P \equiv -\frac{1}{q_c - q_o}\int\limits_c^o z\frac{\partial q}{\partial z}dz \approx -\frac{1}{q_a - q_{o'}}\int\limits_a^{o'} z\frac{\partial q}{\partial z}dz, \tag{20}$$

$$\overline{V^2} \equiv -\frac{1}{q_c - q_o}\int\limits_c^o V^2\frac{\partial q}{\partial r}dr \approx -\frac{1}{q_a - q_{o'}}\int\limits_a^{o'} V^2\frac{\partial q}{\partial r}dr. \tag{21}$$

In (19) α, p, V, r, q are evaluated at point $z = h_b$, $r_a = r_c$ and q_o, V_o are evaluated at $r = r_o$, $z_o = z_{o'}$. Equation (19) summarizes the energy budget of an infinitely narrow cycle $a - c - o - o' - a$ and thus provides a relationship between local variables.

The left-hand side of Eq. (19) represents local heat input per unit mass of moist air (e.g., Gill, 1982, Eq. 3.8.2) multiplied by the thermodynamic efficiency of the cycle ε. While in E-PI $\varepsilon = \varepsilon_C$, Eq. (19) is valid more generally for any $\varepsilon \leq \varepsilon_C$.

The first two terms in the right-hand side of (19) represent turbulence $-\mathbf{f} \cdot d\mathbf{l}$ if $a - c$ is a streamline such that the Bernoulli equation (11) applies. The third term (the one with V_o) accounts for the kinetic energy increment along $o - o'$ discussed in the previous section, see Eq. (10).

The last term in parentheses in the right-hand side of (19) represents potential and kinetic energy increments associated with phase transitions. Term gH_P in (19) accounts for the gravitational power of precipitation; H_P is the mean height where condensation occurs. The meaning of this term is that gas (water vapor) arises at the surface and disappears (i.e. condenses), together with its potential energy, at altitude H_P. This potential energy later dissipates as the falling droplets interact with the air. However, this term in the power budget would be present even if there were no droplets and the condensed gas just disappeared from the atmosphere. We emphasize that, following mass conservation, the gravitational power of precipitation relates to evaporation, i.e. $\partial q/\partial r$ – i.e. the atmosphere must raise newly evaporated water vapor.

The term $(V^2 - \overline{V^2})/2$ accounts for the fact that evaporating water vapor is formally added to the air mixture with velocity V of the air at $a - c$, and then it disappears during condensation with velocity $\overline{V} < V$ in the upper atmosphere. This term can be neglected[2] as $V^2/2$ is small compared to gH_P: since typical mean condensation heights in the tropics are about $H_P \sim 5$ km (Makarieva et al., 2013a, their Fig. 1), with $V \sim 60$ m s^{-1}, $V^2/2$ is only about 4% of gH_P. In what is to follow the term proportional to $(V^2 - \overline{V^2})/2$ is neglected in (19).

2.3 Transition from work to power

To estimate velocity in E-PI a transition is made from work to power and from heat increment to heat flux. This can be done by multiplying (19) by ρ, replacing $\partial/\partial r$ by material derivative d/dt,

$$\frac{d}{dt} \equiv \mathbf{v} \cdot \nabla, \tag{22}$$

[2]In fact, term $(V^2 - \overline{V^2})/2$ can be explicitly accounted for by specifying the interaction between condensate and air (i.e. introducing a specific term to the equations of motion and Bernoulli equation). If condensate is assumed to have the same horizontal velocity as air (see, e.g., Ooyama, 2001; Makarieva et al., 2017b), then as it leaves the air at the surface it has the same velocity as the newly evaporated water vapor and thus net impact of this term to the power budget will be zero. An explicit account of this effect is somewhat lengthy and as it is of minor importance to our derivations (since $V^2/2 \ll gH_P$ in any case) we don't present it (but see Makarieva et al., 2017c, Fig. 1).

and applying the Bernoulli equation to replace the first two terms in the right-hand part of (19) by $-\rho\mathbf{f}\cdot d\mathbf{l}/dt = -\rho\mathbf{f}\cdot\mathbf{v}$ – the power of turbulent friction force per unit volume.

In thus modified Eq. (19),

$$\varepsilon_C\left(-\frac{dp}{dt}+\frac{L\rho}{1+q}\frac{dq}{dt}\right)=\underbrace{-\frac{dp}{dt}-\frac{\rho}{2}\frac{dV^2}{dt}}_{-\rho\mathbf{f}\cdot\mathbf{v}}-\frac{1+q_o}{1+q}\frac{\rho}{2}\frac{dV_o^2}{dt}+\frac{\rho}{1+q}gH_P\frac{dq}{dt}, \tag{23}$$

the following logic is further used, cf. Eqs. (2) and (3):

$$\left(-\frac{dp}{dt}+\frac{L\rho}{1+q}\frac{dq}{dt}\right)h=\rho C_k V(k_s^*-k)=\rho C_k V(c_p\Delta T+L\Delta q), \tag{24}$$

$$\left(-\rho\mathbf{f}\cdot\mathbf{v}-\frac{1+q_o}{1+q}\frac{\rho}{2}\frac{dV_o^2}{dt}\right)h=\rho C_D V^3. \tag{25}$$

Relationship (24) assumes that the local flux of heat from the ocean via the oceanic surface is absorbed by air parcels moving along the isothermal trajectory $a-c$ which has a local thickness h.

Relationship (25) assumes that the rate of kinetic energy change at $o-o'$ is either negligible or arises as part of turbulent power in the boundary layer – in this manner the cycle would be energetically closed. The residual between total turbulent power $-\rho\mathbf{f}\cdot\mathbf{v}$ and the rate of kinetic energy change at $o-o'$ is assumed to be equal to turbulent dissipation rate in the boundary layer – a positive definite flux parameterized as $\rho C_D V^3$ (W m^{-2}) (Bister and Emanuel, 1998, Eq. 6). Tang and Emanuel (2012) emphasized that in E-PI the scale height h for the turbulent fluxes of heat and momentum is assumed to be the same.

If the gravitational power of precipitation is neglected in (23), then Eqs. (23)-(25) yield E-PI formula (4) for maximum hurricane velocity.

Smith et al. (2008) noted that E-PI presumes gradient wind balance in the boundary layer. As we can see, the key equations (19) and (23) do not make this assumption. However, as also noted by Tang and Emanuel (2012), going from (19) to heat fluxes (24) and dissipation power (25) does extrapolate Eq. (19) to the boundary layer (where the gradient wind balance does not hold) – by assuming that the thickness of the isothermal streamline h coincides with the boundary layer. Indeed, already in the original formulation of E-PI an equation identical to (19), which is valid for $z=h$, was obtained for $z=0$ by explicitly assuming the gradient wind balance at $z=0$ (cf. Eqs. (18) and (63) of Emanuel, 1986). As estimated by Tang and Emanuel (2012), the inaccuracy introduced by this extrapolation is around 10-15%, which is approximately what is found in numerical models (e.g., Wang and Xu, 2010; Frisius et al., 2013; Kowaleski and Evans, 2016).

2.4 Sensible heat, latent heat and dissipative heating

The first law of thermodynamics in the form (15) indicates how much heat should be added to the expanding air for it to remain isothermal and increase its moisture content while performing work. It does not specify the source of this heat but only its magnitude – thus the left-hand part of Eq. (24) does not inform us about the sources of heat for the expanding air. Meanwhile the right-hand part of this equation specifies that this heat derives from the ocean in the form of latent heat

$$J_L\equiv\frac{\rho L}{1+q}\frac{dq}{dt}h=\rho C_k V L\Delta q \tag{26}$$

and sensible heat

$$J_S \equiv -\frac{dp}{dt}h = \rho C_k V c_p \Delta T. \tag{27}$$

Bister and Emanuel (1998) proposed that besides the ocean there is another source of heat – turbulent dissipation of kinetic energy of the storm – and suggested that it should be added to the energy budget of the hurricane. However, Eqs. (26) and (27) indicate that dissipative heating cannot be added without reducing sensible heat flux by a similar amount (as previously noted by Makarieva et al., 2010).

Indeed, assuming that $\partial V/\partial r = 0$ at the radius of maximum wind, we find from Bernoulli equation (11)

$$-\rho \mathbf{f} \cdot \mathbf{v} = -\frac{dp}{dt}, \tag{28}$$

i.e. that turbulent dissipation rate per unit area $-\rho h \mathbf{f} \cdot \mathbf{v}$ equals $(-dp/dt)h$ and thus formally coincides with the sensible heat flux (27). The reason is that the sensible heat flux keeps the air isothermal by compensating for the loss of internal energy of air that would otherwise occur as the air performs work. At the radius of maximum wind the dissipation rate equals the rate of this work. Thus dissipative heating from turbulence would exactly compensate for this possible loss of internal energy to work – and thus sensible heat is no longer required nor can it be accomodated by the air parcel without its temperature rising[3]. In other words, according to the first law of thermodynamics an air parcel expanding isothermally at a rate $-dp/dt$ (W m^{-3}) must receive heat at the same rate. If this heat comes from turbulent dissipation, it cannot come from the ocean, and vice versa. Thus E-PI constrains *the sum* of sensible flux and dissipative heating at the radius of maximum wind. (This agrees with the conclusions of Kieu (2015) who showed that the dissipative heating is inherently included in the power budget of the hurricane and cannot be treated as a separate heat source.[4])

Combining Eqs. (28), (27) and (23), where we neglect the term with V_o, which, according to Bister and Emanuel (1998), is usually small unless r_o is very large, we obtain

$$\varepsilon_C(D + J_L) = D + gH_P E \tag{29}$$

and

$$D = \frac{\varepsilon_C - gH_P/L}{1 - \varepsilon_C} J_L. \tag{30}$$

Here $E \equiv J_L/L$ (kg m^{-2} s^{-1}) is the local flux of evaporation. Comparing Eq. (30) with Eq. (21) of Bister and Emanuel (1998) we can see that, the gravitational term aside, the two equations coincide if and only if $J_L = J$, i.e. if total oceanic flux coincides with latent flux (and sensible heating is zero).

[3] Air warming from dissipative heating occurs in numerical models where dissipative heating is included as an additional term in the power budget. For example, Zhang and Altshuler (1999) demonstrated that including this term makes the air warmer than the ocean producing such exotic processes as a large negative flux of sensible heat at peak hurricane intensity. (This contradicts observations.) The increase in maximum velocity in such models arises from a larger Δq rather than a larger efficiency ($T_s/T_o - 1$ instead of $1 - T_o/T_s$).

[4] Unlike in the present work, Kieu (2015, see his Eq. 13) did not analyze the local equations of E-PI but used integral equations for the hurricane as a whole.

In the general case, Eq. (30) shows that irrespective of whether turbulent dissipation contributes to heat balance (or turbulent energy is exported at the level of small eddies), i.e. irrespective of whether the dissipative heating exists or not, Eq. (30) relates turbulent dissipation to oceanic *latent* heat flux at the radius of maximum winds, i.e. where $\partial V/\partial r = 0$. This constitutes a remarkable and convenient feature of E-PI: it is independent of the parameter ΔT governing the sensible heat flux from the ocean. Neither ΔT, nor sensible versus latent heat fluxes have previously been assessed theoretically within the E-PI framework. Note that where $\partial V/\partial r \neq 0$, Eq. (28) and, hence, Eq. (30) are invalid.

3 Velocity estimate

3.1 Correction due to lifting water

Our revised E-PI estimate of maximum velocity follows from (30), (26) and (3):

$$V_{\max}^2 = \frac{\varepsilon_C - gH_P/L}{1-\varepsilon_C}\frac{C_k}{C_D}L\Delta q. \tag{31}$$

Mean condensation height H_P (20) can be calculated from the equation of moist adiabat and depends on surface temperature, the incompleteness of condensation (i.e. altitude where condensation ceases) and, to a lesser degree, on surface relative humidity. In the tropics H_P does not exceed 6 km (see Makarieva et al., 2013a, their Fig. 1).

With $H_P = 6$ km we have $gH_P/L = 0.024$ in (31). This means that for a typical $\varepsilon_C \approx 0.3$ the gravitational power of precipitation reduces $V_{\max}$ by approximately $(gH_P/L)/\varepsilon_C/2 \approx 0.04$, i.e. by 4% at maximum[5] (but this correction would grow with decreasing ε_C). This contrasts the results of Sabuwala et al. (2015) who found that such a reduction can reach as much as 30%, with an average of 20%.

Sabuwala et al. (2015, their Eq. (2), the "adiabatic case") based their analysis on the equation

$$\varepsilon(D+J) = D + gH_P P, \tag{32}$$

which gives

$$D = \frac{\varepsilon_C - gH_P/L(PL/J)}{1-\varepsilon_C}J. \tag{33}$$

Here P (kg m^{-2} s^{-1}) is the local precipitation in the region of maximum wind[6].

Comparing Eqs. (32) and (33) to (29) and (30) we can see that Sabuwala et al. (2015) used local precipitation P instead of local evaporation E and total heat flux J instead of latent flux J_L. The latter difference did not considerably affect the relative magnitude of the correction, but the former one did. Replacing E by P in (32) as compared to (29) leads to the appearance of

[5] Dr. Kerry Emanuel (pers. comm.) suggested that the ultimate correction would be even smaller if one takes into account the heat associated with dissipation of falling hydrometeors.

[6] Equation (33) is obtained from Eq. (2) of Sabuwala et al. (2015) and their additional equation $\dot{Q}_{in} - \dot{Q}_{out} = P$. Note the following differences in notations between Sabuwala et al. (2015) and the present work: $T_s \to T_a$, $\dot{Q}_{in} \to J$, $\dot{Q}_d \to D$, $P \to W_P = gH_P P$ (for the last relationship see Pauluis et al., 2000; Makarieva et al., 2013a).

a large factor $PL/J \gg 1$ at the gravitational term gH_P/L in (33) as compared to (30). For typical Bowen ratios in hurricanes $B \equiv J_S/J_L \approx 1/3$ (e.g., Jaimes et al., 2015) we have $J = (1+B)J_L = (1+B)EL$ and $PL/J = (P/E)/(1+B)$.

Ratio P/E between local precipitaton and evaporation in the region of maximum winds is variable but on average of the order of 10 (see Makarieva et al., 2017a, their Table 1 and Figs. 2 and 3). The corresponding correction to $V_{\max}$ of Sabuwala et al. (2015) is $10(gH_P/L)/[2(1+B)\varepsilon_C] \approx 0.3$, which is almost an order of magnitude larger than in our Eq. (31), i.e. about 30%.

Sabuwala et al. (2015) did not derive their formulations from the orignial assumptions of E-PI. They added what appeared to be a plausible term describing the gravitational power of precipitation to the hurricane's power budget. However, as our analysis has shown, the unjustified replacement of evaporation E by precipitation P caused the estimate of Sabuwala et al. (2015) be too high.

Why does the local power budget of a hurricane in E-PI include evaporation rather than rainfall? A major part of water precipitating within the hurricane is imported from outside (Makarieva et al., 2017a). The E-PI does not explicitly account for this imported moisture. It views the hurricane as a steady-state thermodynamic cycle with all moisture provided locally by evaporation from the ocean. This imported moisture is, however, implicitly accounted for by considering the hypothetical adiabat $o' - a$ to be part of the hurricane's thermodynamic cycle (Fig. 1).

Along $o' - a$ the moisture content of the hypothetically descending air rises by over two orders of magnitude. Most moisture condensing within the hurricane precipitates and cannot serve as a source of water vapor for the descending air. The vertical distribution of humidity along the $o' - a$ path is in fact provided by evaporation and convection outside the storm. These moisture is gathered by the hurricane as it moves through the atmosphere. As the moisture is imported with its own gravitational energy, the storm spends no energy to raise this imported water. The storm only has to raise locally evaporated water, which is why it is evaporation E that enters the power budget equation (29). Notably, Emanuel (1988) indicated that the term accounting for water lifting energy is proportional to the difference in the water profiles of the air rising along $c - o$ path and the environmental air along the hypothetical path $o' - a$ (Emanuel, 1988, see Appendix C and Eq. (C12)). This difference indeed is provided by evaporation along the path $a - c$.

3.2 Scaling of maximum velocity

The magnitude of Δq in Eq. (31), as a measure of thermodynamic disequilibrium between the atmosphere and the ocean, cannot be retrieved from the consideration of the Carnot cycle. Its specification requires an additional assumption. In E-PI it is assumed that relative humidity $\mathcal{H}$ of surface air at the radius of maximum winds is equal to its ambient value $\mathcal{H}_a$ (e.g., Emanuel, 1995, p. 3971):

$$\Delta q = (1 - \mathcal{H}_a) q^*_{sc}, \tag{34}$$

where low index c specifies that the saturated mixing ratio q^*_s, corrresponding to sea surface temperature, is evaluated at radius r_c of maximum winds (Fig. 1).

From the definition of q and the ideal gas law

$$q \equiv \frac{\rho_v}{\rho_d} = \frac{M_v}{M_d}\frac{p_v}{p_d} = (1+q)\frac{M_v}{M}\frac{p_v}{p} = (1+q)\frac{M_v}{M}\mathcal{H}\frac{p_v^*}{p}, \tag{35}$$

where M_v, M_d and M are the molar masses of water vapor, dry air and air as a whole, p_v^* is the saturated partial pressure of water vapor, we have (for derivation details see Makarieva et al., 2017a, Eq. 3):

$$\frac{dq}{q} = \frac{1}{1-\gamma}\left(\frac{d\mathcal{H}}{\mathcal{H}} - \frac{dp}{p} + \frac{dp_v^*}{p_v^*}\right) = (1+q)\frac{M_d}{M}\left(\frac{d\mathcal{H}}{\mathcal{H}} - \frac{dp}{p} + \frac{\mathcal{L}}{RT}\frac{dT}{T}\right), \tag{36}$$

where $\gamma \equiv p_v/p$, $\mathcal{L} = LM_v = 45$ kJ mol^{-1} is molar heat of vaporization, $R = 8.3$ J K^{-1} mol^{-1} is the universal gas constant. The difference between the saturated mixing ratio ($\mathcal{H}_c = 100\%$) at the radius of maximum wind and the ambient mixing ratio ($\mathcal{H}_a < 100\%$) along an isothermal surface where the saturated pressure of the water vapor is constant (and hence the third term in (36) is zero) is a sum of the relative differences of humidity and pressure. The difference in humidity usually dominates. For a typical hurricane with a total pressure drop about 50 hPa of which 10 hPa correspond to the eye, the relative pressure difference between the radius of maximum wind and the ambient pressure of 1010 hPa will be around $\delta p/p \approx 0.04$, which is about one fifth of what the relative humidity contributes, $\delta\mathcal{H}/\mathcal{H} \approx 0.2$ for $\mathcal{H}_a = 80\%$. Neglecting the pressure contribution, $\Delta q \approx \Delta q_a = (1-\mathcal{H}_a)q_{sa}^*$, the main velocity scale in E-PI is stated to depend on the water vapor disequilibrium at the surface in the ambient environment (e.g., Emanuel, 1989, Eq. 38):

$$V_{\max}^2 \sim \frac{\varepsilon}{1-\varepsilon}L(1-\mathcal{H}_a)q_{sa}^*. \tag{37}$$

Compared to (31), Eq. (37) neglects the gravitational power of precipitation and assumes $C_k/C_D \approx 1$. Compared to Eq. (38) of Emanuel (1989), Eq. (37) contains factor $1/(1-\varepsilon)$.

Using the ideal gas law and Eq. (35) we can re-write Eq. (37) as

$$V_{\max}^2 \sim \left(\frac{1}{2}\frac{\varepsilon}{1-\varepsilon}\frac{\mathcal{L}}{RT_s}\frac{1-\mathcal{H}_a}{\mathcal{H}_a}\right)\frac{2p_{vsa}}{\rho_{sa}}, \tag{38}$$

where $p_{vsa} = \mathcal{H}_a p_{vsa}^*$ is the actual partial pressure of water vapor in surface air in the ambient environment, p_{vsa}^* is the saturated partial pressure of water vapor at sea surface temperature in the ambient environment, ρ_{sa} is ambient air density at the surface.

Using typical tropical values $T_s = 300$ K, $\mathcal{H}_a = 0.8$, $\rho_{sa} \approx 1.2$ kg m^{-3}, $\varepsilon = 0.32$ (see, e.g., Table 1 of Emanuel, 1989), we have $p_{vs} = 28$ hPa and $V_{\max} = 70$ m s^{-1}. Neglecting the factor $1/(1-\varepsilon)$ (which is not taken into account in the scaling relationships like Eq. (38) of Emanuel (1989) or Eq. (8) of Emanuel (1997)), we obtain $V_{\max} \approx 60$ m s^{-1} in agreement with Table 1 of Emanuel (1989).

Here our main point is that the coefficient in parentheses in Eq. (38) for the same typical parameters is very close to unity (to be precise, 1.06):

$$\frac{1}{2}\frac{\varepsilon}{1-\varepsilon}\frac{\mathcal{L}}{RT_s}\frac{1-\mathcal{H}_a}{\mathcal{H}_a} \approx 1. \tag{39}$$

This means that numerically the scaling of maximum velocity in E-PI practically coincides with the scaling

$$\rho\frac{V_{\max}^2}{2} = p_{vsa} \tag{40}$$

proposed within the concept of condensation-induced atmospheric dynamics (Makarieva and Gorshkov, 2009, 2011; Makarieva et al., 2013b, 2017a). Moreover, the exponential dependence on surface temperature is captured within the term p_{vs} that is common both to (38) and (40). Unlike p_{vs}, the factor (39) of E-PI depends only weakly on T_s.

While under typical tropical conditions Eqs. (38) and (40) are numerically similar, their physical interpretations differ. In E-PI the key parameter is the disequilibrium between the atmosphere and the ocean, a measure of which is $1 - \mathcal{H}$. In condensation-induced dynamics partial pressure of water vapor is interpreted as the local store of available potential energy which can be converted to the kinetic energy of winds upon condensation. The predictions also differ. The disequilibrium with the sea grows in a drier atmosphere (lower relative humidity) implying more intense hurricanes. Equation (37) predicts that in a drier atmosphere with a lower p_v the storms will be less intense[7]. At the same time the mechanisms underlying Eqs. (38) and (40) are not necessarily mutually exclusive since a low relative humidity $\mathcal{H} < 1$ at the surface is ultimately a consequence of condensation and moisture removal from the ascending air. The physical meaning of the relationship (39) remains to be investigated.

4 Discussion

4.1 The dynamic interpretation of E-PI

The starting point of the E-PI concept in its original formulation was to assume that surfaces of constant angular momentum are also surfaces of constant moist entropy (Emanuel, 1986). This key assumption linked the dynamic and thermodynamic aspects of the concept. We have shown that it is possible to derive E-PI without considering entropy and separating dynamics from thermodynamics.

Neglecting energy expended to lift water, the logic of E-PI can be summarized as follows. First, the total work of the cycle, $A^+ + A^-$, is equal to ε_C times the isothermal heat input from the ocean. Second, from the first law of thermodynamics this isothermal heat input is the sum of A^+ (work in the boundary layer) and latent heat input. Third, from the Bernoulli equation work A^+ on a horizontal streamline is equal to the radial increment of kinetic energy minus turbulent friction losses, Eq. (12). Forth, E-PI assumes that A^- (work above the boundary layer) is negative and (approximately) equal in magnitude to the positive radial increment of kinetic energy in the boundary layer, Eq. (13). These four statements relate the radial increment of kinetic energy, turbulent friction losses and latent heat input. By definition, at the radius of maximum wind the radial increment of kinetic energy is zero[8]. Thus at the radius of maximum wind E-PI yields a relationship between local latent heat input and

[7] An extreme example is polar cyclones which develop central pressure deficits of only a few hectopascals mirroring the low moisture content of the polar atmosphere (Makarieva et al., 2013b).

[8] This assumption, $V_c^2 - V_a^2 = 0$, was originally used by Emanuel (1986) for a non-local path $a - c$ for his estimate of the pressure difference $p_c - p_a$, where point c corresponded to the hurricane center. Emanuel (1997) revised this derivation considering the hurricane eye as a separate streamline from that of the converging air.

turbulent friction, Eq. (30), from which PI maximum velocity derives, Eq. (31). This logic works when the kinetic energy increment along the $o-o'$ path, $(V_o^2 - V_{o'}^2)/2$, can be neglected compared to the work $\alpha(p_a - p_c)$.

Thermodynamics enters this picture only as parameter ε_C, which applies when the considered cycle is a Carnot cycle. If it is not, then a lower $\varepsilon < \varepsilon_C$ should be used. For example, if $o'-a$ and $c-o$ are not reversible adiabates but moist pseudoadiabates, one would have to replace ε_C in (19) by a smaller value. Another example: our derivations assumed that $o-o'$ is simultaneously a vertical ($r_o = r_{o'}$) and an isotherm ($T_o = T_{o'}$). If, however, the vertical path connecting the two adiabates in Fig. (1) is not an isotherm, the coefficient of converting heat to work in (17) and (19) should again be less than $\varepsilon_C = (T_a - T_o)/T_a$. Tang and Emanuel (2012) discussed alternative constraints that can be applied to the processes at $o-o'$ instead of isothermy. In any case, the key dynamic proposition of E-PI, Eq. (13), follows from the geostrophic and hydrostatic balance. This relationship applies to any steady-state circulation pattern where these balances hold irrespective of thermodynamics.

4.2 Surface heat fluxes

Our analysis resolves ambiguity concerning the role of sensible heat in E-PI. This ambiguity appears to have escaped previous attention. According to observations, sensible heat in intense tropical storms in the region of maximum wind constitutes around one quarter of the total heat flux. This implies a temperature difference ΔT of a few degrees Kelvin between the sea surface and the adjacent air. This difference is indeed observed (e.g., Jaimes et al., 2015, Fig. 9c). With relative humidity close to unity, namely this temperature difference accounts not only for the sensible but also for the latent heat input from the ocean.

Since sensible heat has never been explicitly discussed within E-PI, in the literature there is some confusion on this matter. For example, Holland (1997, Table 1) believes that in E-PI the sea surface temperature equals air temperature ($\Delta T = 0$), while according to Garner (2015) E-PI assumes a surface air-temperature deficit of 1°-2°C. The assumption $\Delta T = 0$ is logically suggested by the first two unnumbered equations on p. 589, right column of Emanuel (1986). These equations specify that temperature T_s of air at $z = 0$ (at the top of the surface layer) is independent of distance from the hurricane center, i.e. that the surface air is isothermal. The assumption $T_s(r) = const$ implies that T_s is set by the isothermal sea surface and coincides with the sea surface temperature. The reason is that there is no definite value for a fixed temperature difference between the sea surface and the adjacent air (and none is discussed in the theoretical foundations of E-PI). Indeed, the assumption of $\Delta T = 0$ was adopted by Emanuel (1986, p. 591) in his first comparison of E-PI with empirical data.

Besides observations showing $\Delta T \neq 0$, the air isothermal at $z = 0$ is not compatible with the other original assumptions of E-PI. The derivations of Emanuel (1986) presume that 1) that air at $z = h_b$ (height of the boundary layer) is saturated; 2) that it is isothermal and 3) that air at $z = 0$ is also isothermal. An additional assumption 4) of E-PI is that at the radius of maximum wind relative humidity conserves its ambient value. As follows from Eq. (36), this means that moisture content increases as the air moves from a to c (at the expense of $-dp/p > 0$). Had this not been the case, latent heat input into the radially converging air would be zero.

These conditions, air saturated and isothermal at $z = h_b$, air isothermal at $z = 0$ and q at $z = h_b$ growing from a to c are not compatible with each other[9]. Our analysis shows that this problem is irrelevant for the main result of E-PI, since E-PI relates turbulent friction losses to latent heat input independent of sensible heat input (or presence or absence of dissipative heating)[10]. To our knowledge, these relationships between latent heat, sensible heat and dissipative heating have not been described previously. Likely this reflects that the flux of heat has not previously been formulated in terms of dp/dt and dq/dt. Previously all derivations considered potential temperature θ and entropy, which masked some of the relevant physical relationships[11].

Another reason why Eq. (31) was not been previously formulated is because E-PI did not use $\partial V/\partial r = 0$ (the condition for maximum velocity) to derive Eq. (4) (a point noted by Montgomery and Smith, 2017). Putting $\partial V/\partial r = 0$ in (23) does not change the parameterization (25), because according to the Bernoulli equation $-dp/dt - \rho d(V^2/2)/dt$ for a constant z is always equal to $-\rho \mathbf{f} \cdot \mathbf{v}$ irrespective of the magnitude of dV^2/dt. However, as we have shown, see Eq. (28), the condition $\partial V/\partial r = 0$ is essential for an account of external and internal heat flows.

4.3 Scaling of maximum velocity

The E-PI concept is a self-consistent theory that yields the magnitude of storm velocity from environmental parameters. Unlike numerical models of tropical stormes tuned to produce the desired patterns, the E-PI concept seeks to explain why hurricanes have a typical maximum speed of 60 m s^{-1} on the basis of verifiable assumptions. In E-PI the maximum velocity depends on the moisture disequilibrium between the atmosphere and the ocean: the lower the relative humidity, the higher the velocity.

A question faced by the E-PI concept as a theory is the nature of this disequilibrium. How is it maintained? Intense evaporation in the region of maximum winds should have rapidly driven relative humidity to unity, and with $\mathcal{H} = 1$, PI maximum velocity is zero. Indeed, relative humidity in the region of maximum winds in intense storms is close to 100%, conspicuous examples include hurricanes Isabel (Montgomery et al., 2006, Fig. 4d) and Earl (Jaimes et al., 2015, Fig. 9d). For example, in hurricane Isabel relative humidity rose from 75% outside the storm to 97% at the windwall in the vicinity of maximum wind.

With relative humidity close to 100%, moisture input from the ocean occurs due to the lower temperature of the surface air. For example, in Isabel the surface air cooled by 4 K as it moved from the storm's outskirts towards the radius of maximum wind. This cooling is responsible for $\Delta q > 0$ which allows for a positive latent heat flux[12]. But why should the air cool, by

[9] In brief, for the isothermy at both $z = 0$ and $z = h_b$ to be satisfied, the lapse rate below $z = h_b$ must be the same everywhere; i.e. it must be dry adiabatic (since at different q the moist adiabatic lapse rate is not the same). However, saturation height z_1 for a given temperature depends on q: the higher the q, the lower the saturation height (see, e.g., Makarieva et al., 2013a, Fig. 1b-d); when q is saturated, $z_1 = 0$. So, if q increases from a to c, the level at which it becomes saturated diminishes. Thus, if at point c the air is saturated at $z = h_b$ but not below, it cannot be saturated at point a (with a smaller q) at the same height $z = h_b$, thus the first condition is violated.

[10] We have not discussed several other aspects of E-PI, in particular, how the pressure value at the radius of maximum wind is determined, which is necessary to close the problem and obtain an exact estimate of Δq, see Eq. (34). This requires setting one of the two radii (r_c or r_a) and integrating Eq. (A.2) under additional assumptions about the radial behavior of $\mathcal{H}$ (Emanuel, 1986; Tang and Emanuel, 2012).

[11] In this formulism, sensible heat input Q_S for an isothermal process is written not as $Q_S = -\int \alpha_d dp$ but in terms of entropy and the Gibbs function (e.g., Pauluis, 2011, p. 96), which obscures the equivalence between work and sensible heat input.

[12] Another reason for air cooling, as suggested by Dr. Kerry Emanuel (pers. comm.) is the re-evaporation of the falling droplets.

how much and under which circumstances? Namely these factors, and not the ambient relative humidity as Eq. (34) presumes, would determine PI maximum velocity.

One could argue that Eq. (34) is a secondary assumption for E-PI, with the main result being Eq. (31). However, if it is the air cooling that is responsible for moisture input, the question arises how the decreasing temperature in the converging air is compatible with the basic assumptions of E-PI that produced Eq. (31). As we discussed, E-PI requires that within the boundary layer there is a *saturated horizontal isotherm* $z = h_b$ along which the air increases its moisture content towards the hurricane center (Fig. (1). If such a surface exists and for a given r the air below $z = h_b$ is moist adiabatic (as Fig. 1 of Emanuel (1986) assumes), the temperature of surface air must rise from a to c. Conversely, if the surface air temperature declines from a to c, a saturated horizontal isotherm cannot exist. For example, in hurricane Isabel an isothermal surface occurred at $z \approx 1.8$ km, but was not saturated: relative humidity rose from 60% outside the storm to 95% in the windwall (Montgomery et al., 2006, cf. Figs. 4c and 4d).

In other words, to estimate maximum velocity the environmental parameters in E-PI combine differently than was originally assumed. We gave an alternative explanation to the maximum velocity scaling having shown that maximum PI kinetic energy is equal to the partial pressure of water vapor at the surface, Eq. (40). This scaling is central to the concept of condensation-induced atmospheric dynamics (Makarieva et al., 2014, 2015). In the Earth's gravitational field partial pressure of water vapor provides a measure of dynamic disequilibrium, since the air cannot rise adiabatically without water vapor changing state and impacting pressure gradients. For condensation-induced hurricanes, the key process is the positive feedback between the radial air motion and the pressure drop at the surface associated with condensation and hydrostatic adjustment. As air streams towards the hurricane center and ascends, the water vapor condenses and the air pressure drops as determined by the partial pressure of water vapor at the surface.

This concept requires more attention and we value any feedback. One reviewer noted that condensation cannot lead to considerable pressure gradients because droplets that form upon condensation are falling with terminal velocity and thus their weight compensate for all possible pressure drop due to water vapor removal. This would make sense, at least in the vertical dimension, if all condensed moisture remained in the air. However, as discussed by Makarieva et al. (2017b), at the moment a droplet forms it has the same velocity as the air and does not impose any velocity-related force on it. The droplets accelerate relative to the air, and when they reach their terminal velocity, it is so large that most of condensed moisture is removed from the air while it is ascending. Even in hurricanes the amount of condensed moisture remaining in moist air is about one percent of the original water vapor. Thus this residual condensed moisture cannot compensate for the condensation-induced pressure perturbations.

Another reviewer opined that atmospheric moisture content is unlikely be critical for hurricane formation since there are models of "dry hurricanes" driven by heat (not moisture) flux from the ocean. This implies that fluxes of energy should be more important than moisture stores. However, unlike in E-PI concept where the measure of disequilibrium, $1 - \mathcal{H}_a$, is a real atmospheric property, in dry hurricanes the thermal disequilibrium between the air and the ocean is an arbitrary choice for the modeller. This disequilibrium is artificially maintained by setting a prescribed temperature difference ΔT between the air and the ocean (Mrowiec et al., 2011). For an isothermal ocean, this means that the air is isothermal too. In the view of Eq. (27), the existence of a pressure gradient proportional to ΔT is built into such models. However, since a key process – how the heat flux works to destroy the thermal disequilibrium – is neglected in these models, their realism and wider relevance remain undetermined.

Finally, a colleague noted that if, in a model hurricane, surface fluxes are switched off, the storm does not intensify. This does not conflict with our understanding of condensation-induced dynamics, in which storms require moist air to persist. In current models of motionless hurricanes the only source of moisture is the ocean, so if this flux discontinues the atmosphere dries and any moisture-driven would necessarily cease. But in real storms most moisture derives not from concurrent evaporation but frompreviously accumulated water vapor in the atmospheric air that feed into the system (Makarieva et al., 2017a). The motionless model storms lack access to such moisture and depend solely (and artificially) on the ocean.

At the same time, we believe that surface heat fluxes remain relevant to our broader dynamic interpretation of E-PI, that the increment of kinetic energy from a to c must be sufficient for the air to overcome the negative pressure gradient in the upper atmosphere.

The air must have sufficient energy to flow away from the hurricane. If not generated in the boundary layer, this energy could derive from a pressure gradient in the upper atmosphere: if, at the expense of the hurricane's extra warmth, the air pressure in the column above the area of maximum wind is higher than in the ambient environment, this pressure gradient will accelerate the air outward. However, a significant pressure deficit at the surface precludes the formation of a significant pressure surplus aloft (e.g., Makarieva et al., 2017d, Fig. 1d).

Moreover, this pressure deficit is what accelerates air in the boundary layer. If the pressure gradient is sufficiently steep and the radial motion sufficiently rapid, the expansion of air will be accompanied by a drop of temperature (i.e. the process will be closer to an adiabat than to an isotherm). In hurricane Isabel the surface air cooled by about 4 K between the outer core (150-250 km) and the eyewall (40-50 km) (Montgomery et al., 2006, Fig. 4c), while pressure fell from less than 1013 hPa to approx 960 hPa at the eyewall (Aberson et al., 2006, Fig. 4)[13]. This is almost a dry adiabatic process with $dp/p = (C_p/R)dT/T$, where $C_p = (7/2)R$ is molar heat capacity of air at constant pressure. (Likewise Eq. (36) gives $dq/q \approx 0$ with the observed change of relative humidity from 80% to 97% over the same distance.) The surface air streams towards the center so rapidly that it lacks time to take much heat from the ocean.

If the warm air creates a pressure surplus aloft facilitating the outflow, cold air, conversely, creates a pressure deficit. This enhances the pressure gradient in the upper atmosphere against which the air must work to leave the hurricane. Consequently, the storm cannot deepen indefinitely, because at a certain moment *the kinetic energy the adiabatically cooling air acquires at the surface becomes insufficient to overcome the pressure gradient aloft, which grows with cooling*, and the outflow must weaken. This condition provides distinct constraints on storm intensity. Further research is needed to see how this mechanism can be applied to real hurricanes.

[13] Air pressure at the outermost closed isobar ~ 465 km from the center was 1013 hPa, hence at 150-250 km from the center it should have been smaller.

Appendix A: Equivalence between our Eq. (17) and Eq. (18) of Emanuel (1986)

Taking into account that, according to (7) and (8),

$$V^2 = r\alpha\frac{\partial p}{\partial r} - fVr = r\alpha\frac{\partial p}{\partial r} - fM + \frac{f^2r^2}{2}, \tag{A.1}$$

recalling that $M_a = M_{o'}$, $M_c = M_o$, $r_o = r_{o'}$ and assuming, following Emanuel (1986), that for $r_a \le r \le r_o$ the radial pressure gradient is sufficiently small for the first term in the right-hand side of (A.1) to be neglected, we obtain from Eq. (10)

$$-\oint \alpha dp = -\int_a^c \alpha\frac{\partial p}{\partial r}dr - \frac{1}{2}r\alpha\frac{\partial p}{\partial r}\Big|_c - \frac{f^2}{4}\left(r_c^2 - r_a^2\right). \tag{A.2}$$

In the case of E-PI the cycle's efficiency is equal to Carnot efficiency, $\varepsilon_C = (T_a - T_o)/T_a$. Combining (10) and (15) and using (A.2) we find

$$\varepsilon_C\int_a^c\left(-\alpha_d\frac{\partial p}{\partial r} + L\frac{\partial q}{\partial r}\right)dr = -\int_a^c \alpha\frac{\partial p}{\partial r}dr - \frac{1}{2}r\alpha\frac{\partial p}{\partial r}\Big|_c - \frac{f^2}{4}\left(r_c^2 - r_a^2\right) - \oint \alpha q dp. \tag{A.3}$$

Using the definition of equivalent potential temperature $\theta_e = \theta\exp\left(Lq/c_pT\right)$, the Exner function $\pi \equiv (p/p_0)^{R/C_p} = T/\theta$ (Emanuel, 1986, Eq. 15), where θ is the potential temperature and $R/C_p = 2/7$, assuming $\alpha \approx \alpha_d$ is constant, changing the notations $r_c \to r, r_a \to r_0, T_a \to T_B, T_o \to \overline{T}_{\text{out}}$ and neglecting the last term in the right-hand side of (A.3), we obtain Eq. (18) of Emanuel (1986) from Eq. (A.3):

$$-\frac{T_B - \overline{T}_{\text{out}}}{T_B}\ln\frac{\theta_e}{\theta_{ea}} = \ln\frac{\pi}{\pi_a} + \frac{1}{2}r\frac{\partial\ln\pi}{\partial r} + \frac{1}{4}\frac{f^2}{c_pT_B}\left(r^2 - r_0^2\right). \tag{A.4}$$

Appendix B: Deriving Eq. (18)

$$-\oint \alpha q dp = -\oint \alpha q\frac{\partial p}{\partial r}dr + \oint qgdz = -\int_a^c \alpha q\frac{\partial p}{\partial r}dr - \int_c^o \alpha q\frac{\partial p}{\partial r}dr - \int_{o'}^a \alpha q\frac{\partial p}{\partial r}dr - \oint gz\frac{\partial q}{\partial z}dz \tag{B.1}$$

$$= -\int_a^c \alpha q\frac{\partial p}{\partial r}dr - \frac{1}{2}\left(q_cV_c^2 - q_aV_a^2 + q_{o'}V_{o'}^2 - q_oV_o^2\right) - \int_c^o \frac{V^2}{2}\frac{\partial q}{\partial r}dr - \int_{o'}^a \frac{V^2}{2}\frac{\partial q}{\partial r}dr - \oint gz\frac{\partial q}{\partial z}dz \tag{B.2}$$

$$= -\int_a^c q\left(\alpha\frac{\partial p}{\partial r} + \frac{1}{2}\frac{\partial V^2}{\partial r}\right)dr - \frac{1}{2}\left(q_{o'}V_{o'}^2 - q_oV_o^2\right) - \oint \frac{V^2}{2}\frac{\partial q}{\partial r}dr - \oint gz\frac{\partial q}{\partial z}dz \tag{B.3}$$

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